Modeling of cloud microphysics: Can we do better?

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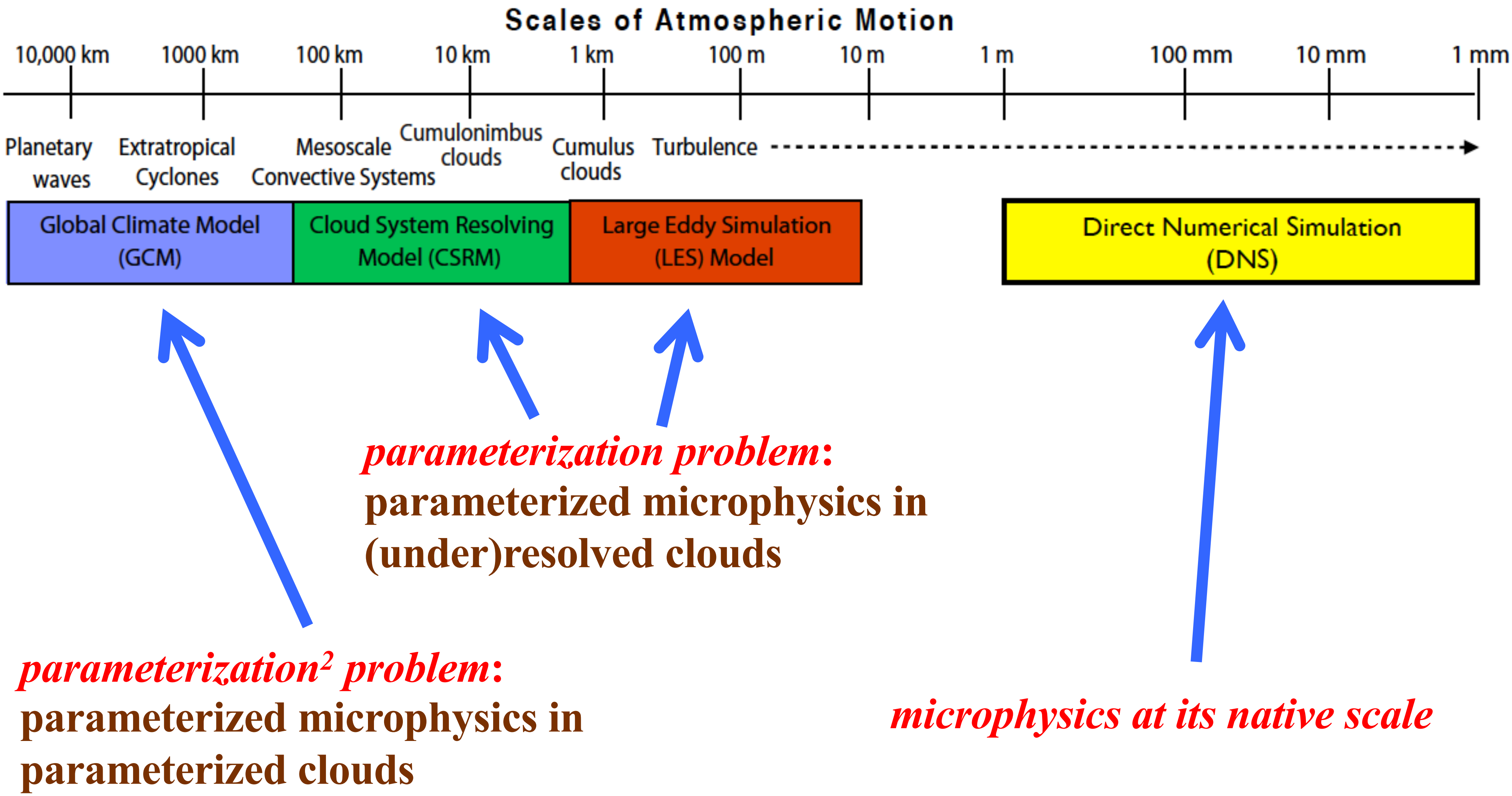
MODELING OF CLOUD MICROPHYSICS

Can We Do Better?

Wojciech W. Grabowski, Hugh Morrison, Shin-Ichiro Shima, Gustavo C. Abade,
Piotr Dziekan, and Hanna Pawlowska

The Lagrangian particle-based approach is an emerging technique to model cloud microphysics and its coupling with dynamics, offering significant advantages over Eulerian approaches typically used in cloud models.

BAMS April 2019 issue



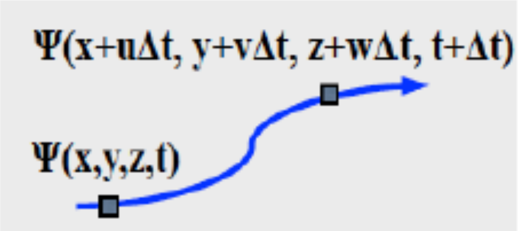
Cloud microphysics across scales

Traditional modeling approaches are based on continuous medium approach, that is, applying density of various condensed water species (e.g., mass of particles per unit volume).

In practice, mixing ratios are used (i.e., mass per unit mass of dry air) as these are conserved along fluid trajectories when particle growth and sedimentation are excluded.

Such an approach has been a workhorse for cloud modeling for decades…

Lagrangian versus Eulerian formulation



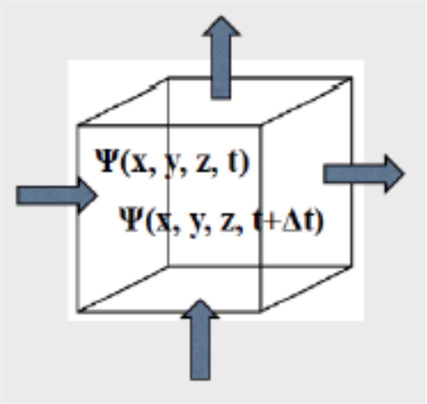
$$\frac{D\Psi}{Dt} = S$$

or

$$\frac{\partial \Psi}{\partial t} + \mathbf{u} \cdot \nabla \Psi = S$$

combined with dry air continuity equation:

$$\frac{\partial \rho_a}{\partial t} + \nabla(\rho_a \mathbf{u}) = 0$$



gives:

$$\frac{\partial \rho_a \Psi}{\partial t} + \nabla(\rho_a \mathbf{u} \Psi) = \rho_a S$$

For the anelastic system:

$$\frac{\partial \Psi}{\partial t} + \frac{1}{\rho_o} \nabla(\rho_o \mathbf{u} \Psi) = S$$

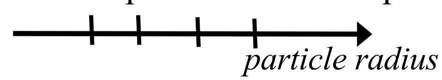
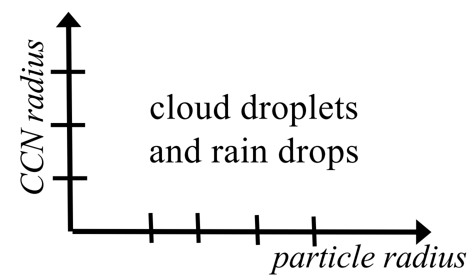
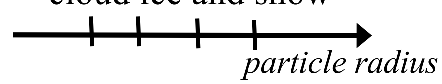
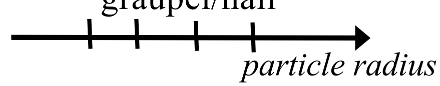
$$\rho_o = \rho_o(z)$$

Microphysical schemes:

Bulk schemes: single-moment (mass only, really no μphysics)
double-moment (mass and number)
triple-moment (mass, number, spectral shape).

Bin schemes: representing the spectrum of particles
(single moment, double moment).

Multidimensional bin schemes: representing the spectrum plus
additional particle properties (e.g., aerosol).

	single-moment bulk microphysics	double-moment bulk microphysics	bin microphysics	2D bin microphysics
		warm-rain microphysics		
cloud droplets	q_c	q_c N_c	cloud droplets and rain drops 	
rain drops	q_r	q_r N_r		
		ice microphysics		
cloud ice	q_i	q_i N_i	cloud ice and snow 	
snow	q_s	q_s N_s		
graupel/hail	q_g	q_g N_g	graupel/hail 	

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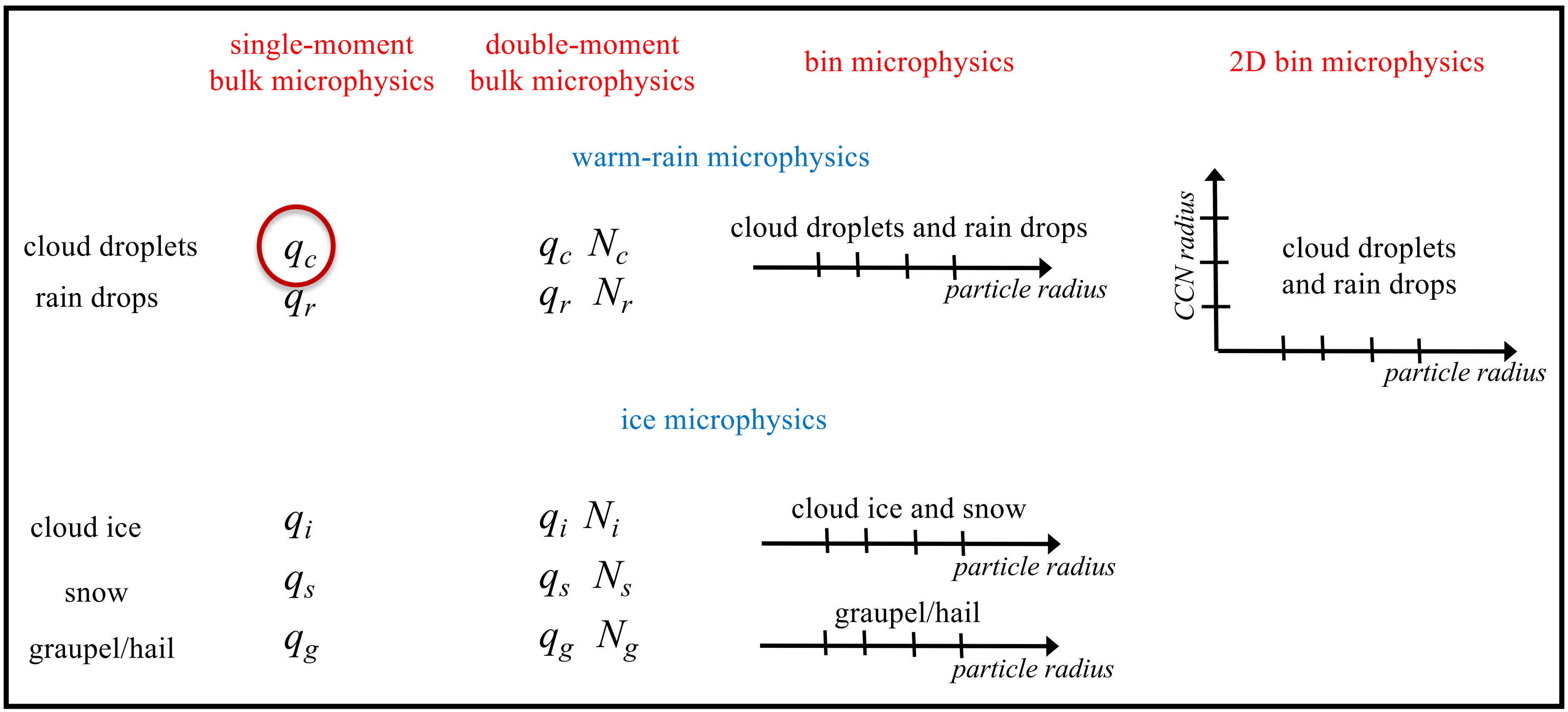
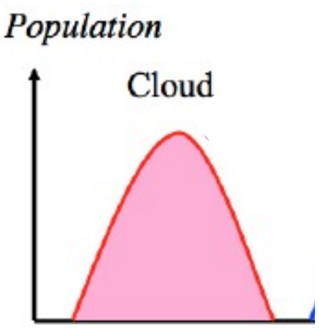


BULK MODEL OF CONDENSATION:



$$\frac{d\theta}{dt} = \frac{L_v\theta}{c_pT}C_d$$

$$\frac{dq_v}{dt} = -C_d$$

$$\frac{dq_c}{dt} = \quad C_d$$

θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud water* mixing ratio
L_v - latent heat of condensation/evaporation
C_d - condensation rate
Note: θ/T function of pressure only ($\approx \theta_o/T_o$)

$$\frac{L_v}{c_p\Pi_e}$$

C_d is defined such that cloud is always at saturation, which is a very good approximation:

$$q_c = 0 \quad \text{if} \quad q_v < q_{vs}$$

$$q_c > 0 \quad \text{only if} \quad q_v = q_{vs}$$

where $q_{vs}(p,T) \approx 0.622\dfrac{e_s(T)}{p}$ is the water vapor mixing ratio at saturation

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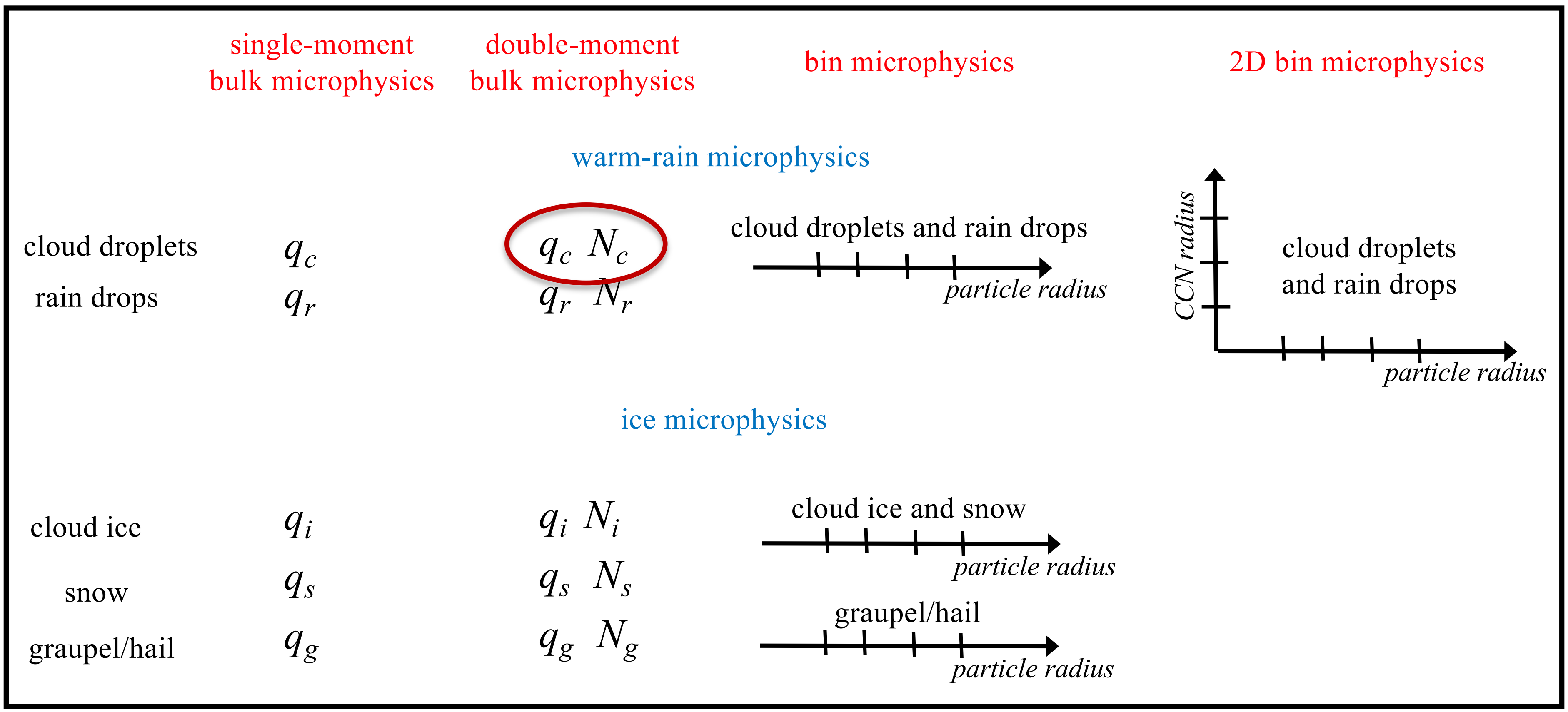

DOUBLE-MOMENT SCHEME:

$$\frac{D\theta}{Dt} = \frac{L_v \theta}{c_p T} C_d$$

$$\frac{Dq_v}{Dt} = -C_d$$

$$\frac{Dq_c}{Dt} = \quad C_d$$

$$\frac{DN_c}{Dt} = \quad S_{act}$$

N_c - number mixing ratio (number od droplets in a unit mass of dry air) of cloud droplets; S_{act} - source of cloud droplets (activation)

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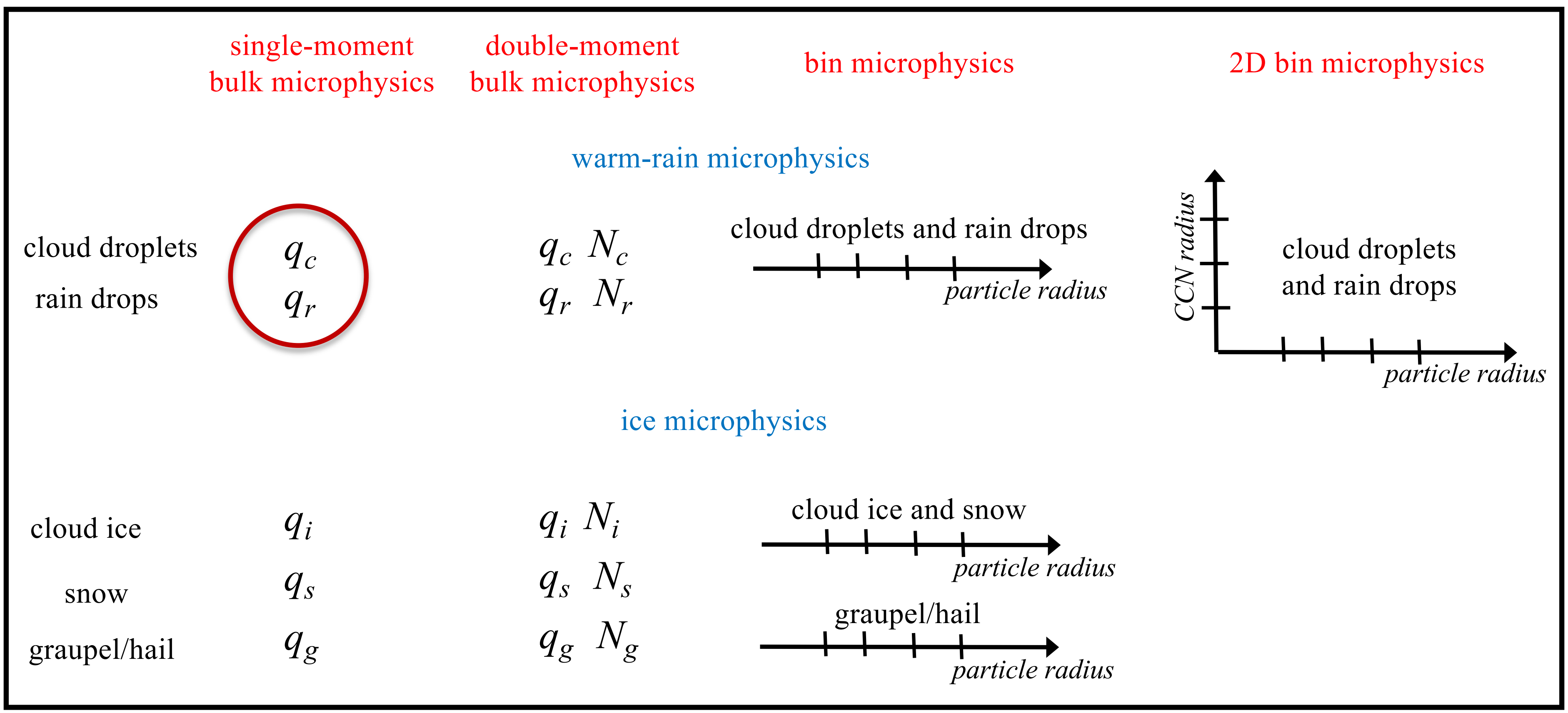
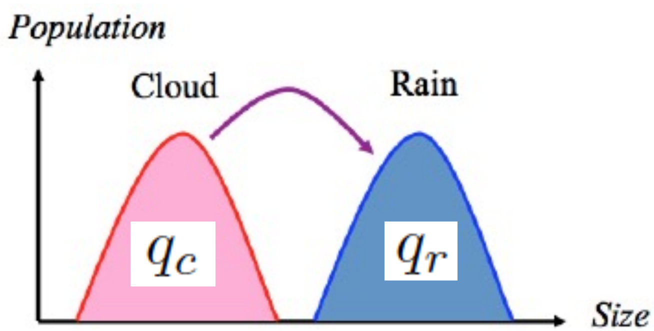


Adding rain or drizzle:

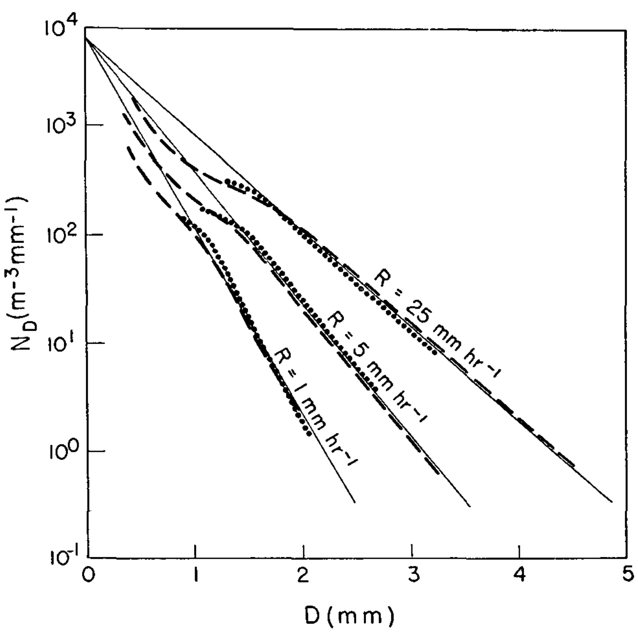


THE DISTRIBUTION OF RAINDROPS WITH SIZE

By J. S. Marshall and W. McK. Palmer[1]

McGill University, Montreal

(Manuscript received 26 January 1948)



WARM RAIN BULK MODEL (Kessler 1969):

$$\frac{D\theta}{Dt} = \frac{L_v\theta}{c_pT}(C_d - EVAP)$$

$$\frac{Dq_v}{Dt} = -C_d + EVAP$$

$$\frac{Dq_c}{Dt} = C_d - AUT - ACC$$

$$\frac{Dq_r}{Dt} = \frac{1}{\rho}\frac{\partial}{\partial z}(\rho q_r v_t) + AUT + ACC - EVAP$$

θ - potential temperature
q_v - *water vapor* mixing ratio
q_c - *cloud water* mixing ratio
q_r - *rain water* mixing ratio
C_d - condensation rate
$EVAP$ - rain evaporation rate
AUT - "autoconversion" rate: $q_c \rightarrow q_r$
ACC - accretion rate: $q_c, q_r \rightarrow q_r$

$v_t(q_r)$ - rain terminal velocity (typically derived by assuming a drop size distribution; e.g., the Marshall-Palmer distribution $N(D) = N_o exp(-\Lambda D)$, $N_o = 10^7\ \mathrm{m}^{-4}$).

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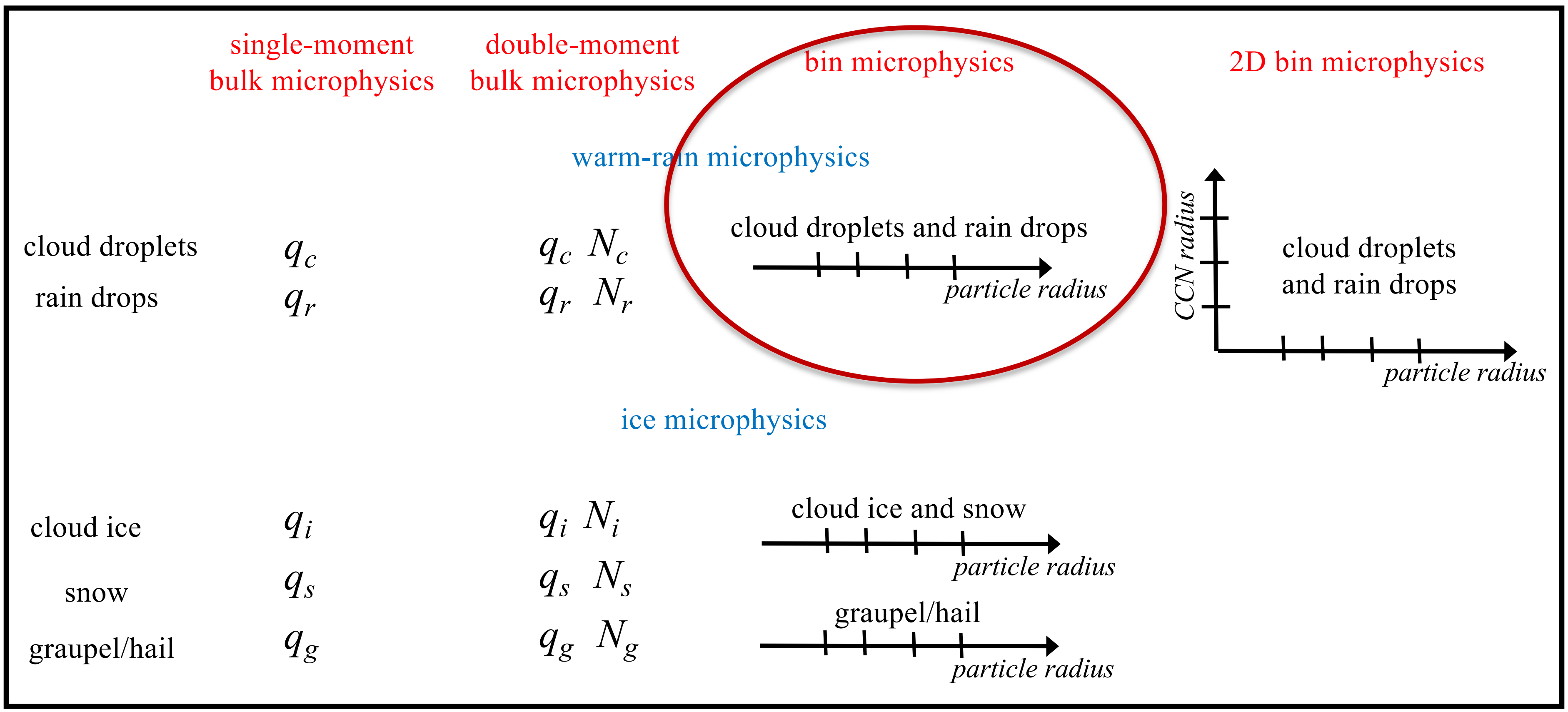
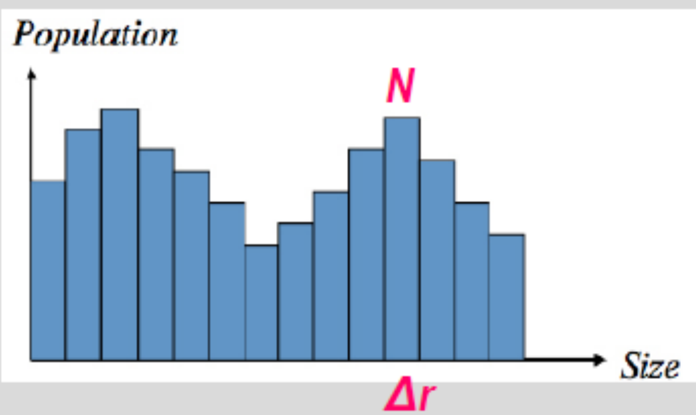


BIN-RESOLVING WARM MICROPHYSICS:

Introducing *spectral density function* $f(r,t)$:

$$f(r,t) \equiv \frac{d\,N(r,t)}{d\,r}$$

$d\,N(r,t)$ is the concentration (per unit mass as mixing ratio) of droplets smaller than r (cumulative concentration).



$$f = N / \Delta r$$

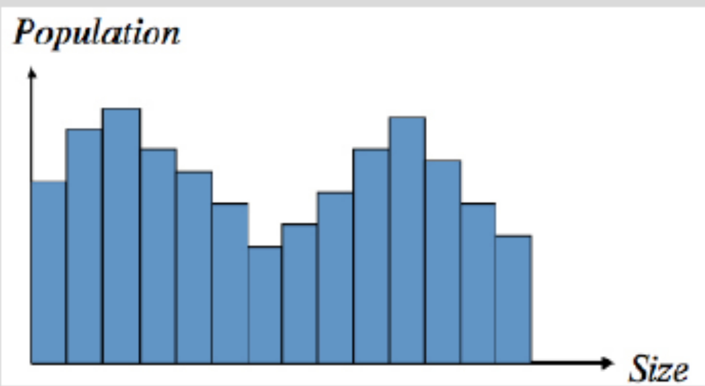
Continuity equation for the growth by condensation:

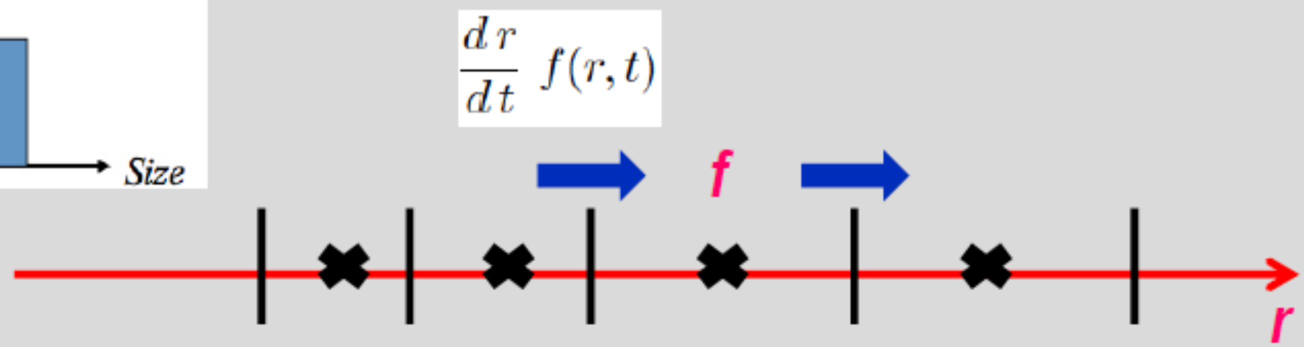
$$\frac{\partial f(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{d\,r}{d\,t}\ f(r,t)\right) = 0$$

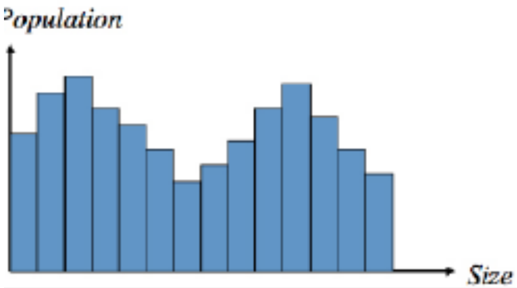
where $\frac{d\,r}{d\,t}$ is growth rate of a droplet with radius r:

$$\frac{d\,r}{d\,t} = \frac{A(T,p)\ S}{r}$$

$S = \frac{q_v}{q_{vs}} - 1$ is the supersaturation; q_v is the ambient water vapor mixing ratio; $q_{vs}(p,T)$ is the saturated water vapor mixing ratio.







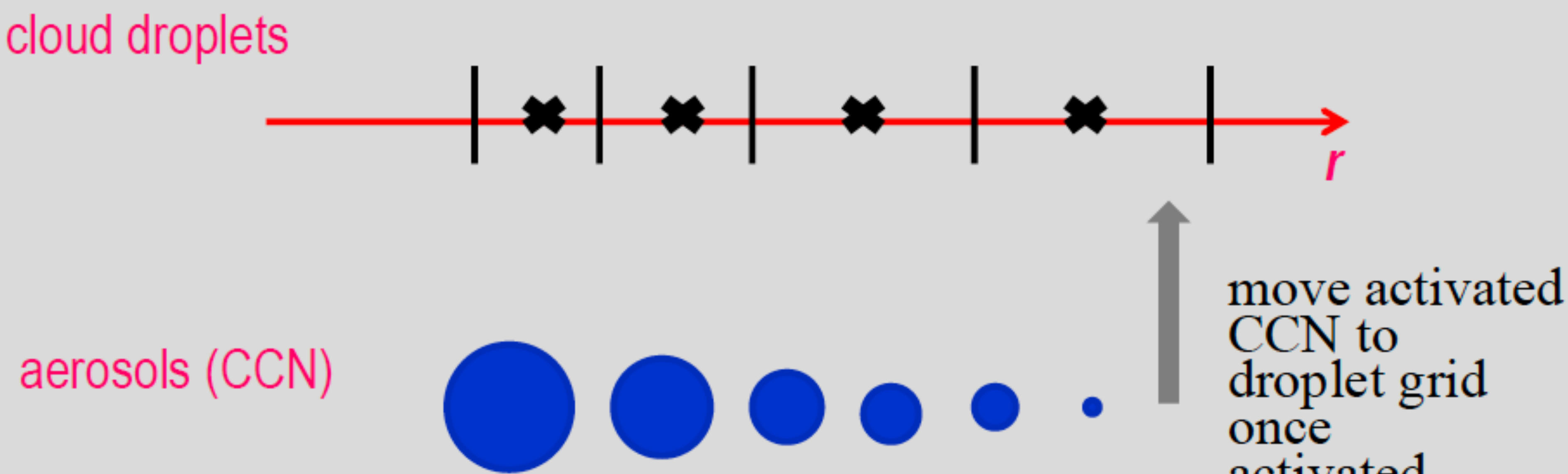
BIN-RESOLVING WARM MICROPHYSICS:

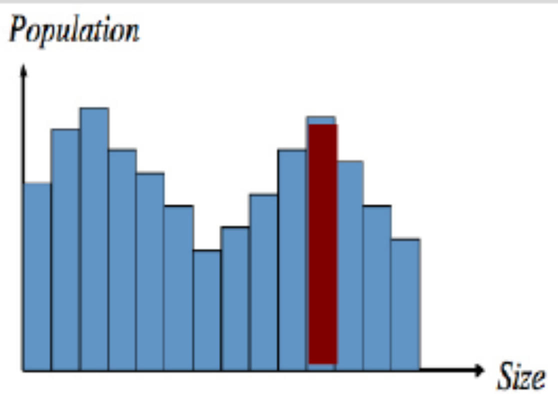
ACTIVATION AND CONDENSATION

Continuity equation for activation and growth by condensation:

$$\frac{\partial f(r,t)}{\partial t} + \frac{\partial}{\partial r}\left(\frac{dr}{dt}\, f(r,t)\right) = S_{nucl}$$

where S_{nucl} is the source associated with activation of cloud droplets (CCN activation).





GROWTH BY COLLISION/COALESCENCE

The Smoluchowski equation (aka *kinetic collection equation*, *stochastic coalescence equation*) for the spectral density function $f(m,t)$:

$$\frac{\partial f(m,t)}{\partial t} =$$

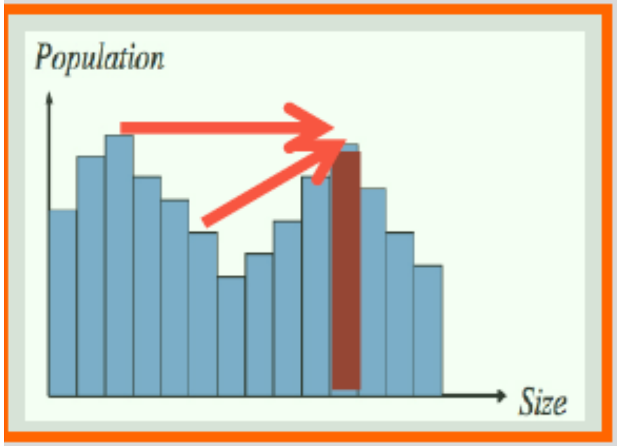
$$= \frac{1}{2}\int_0^m f(m-M,t)\, f(M,t)\, K(m-M,M)\, dM$$

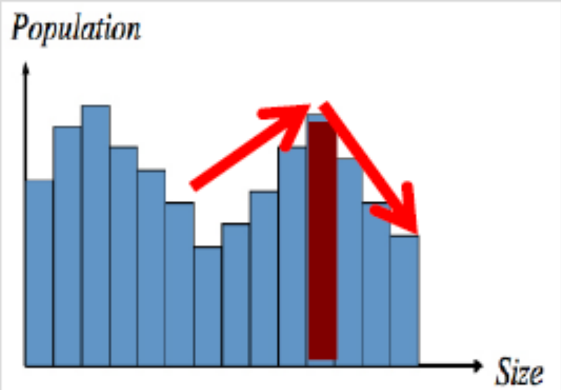
$$- f(m,t) \int_0^\infty f(M,t)\, K(m,M)\, dM$$

m, M - droplet masses

$K(m,M)$ - *collection kernel*; frequency of collisions (per unit volume of air) between droplets with mass m and M

GROWTH BY COLLISION/COALESCENCE





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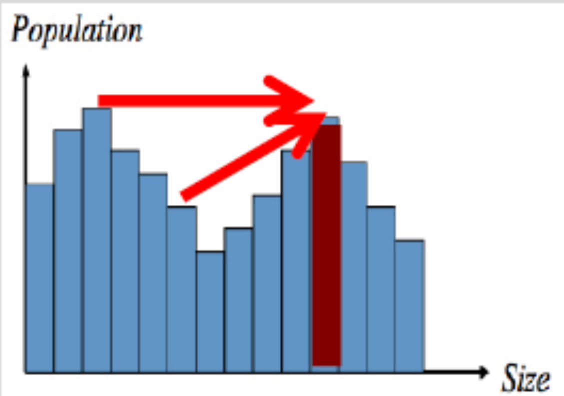

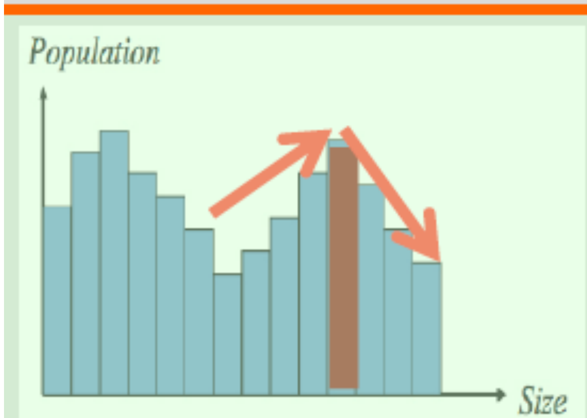

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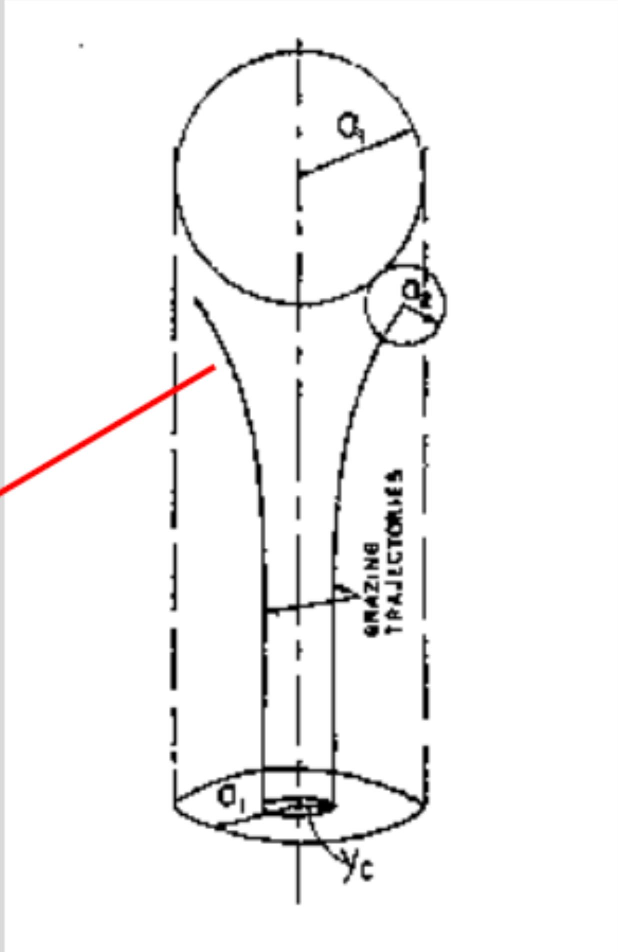
Growth of water droplets by gravitational collision-coalescence:

$$K(m_{a1}, m_{a2}) = E_c\ \pi(a_1 + a_2)^2\ |(V_{a_1} - V_{a_2}|$$



Grazing trajectory

Collision efficiency:

$$E_c = \frac{y_c^2}{(a_1 + a_2)^2}$$

Droplet inertia is the key; without it, there will be no collisions. This is why collision efficiency for droplets smaller than 10 µm is very small.

$$K(m_{a1}, m_{a2}) = E_c \pi (a_1 + a_2)^2 \; |(V_{a_1} - V_{a_2}|$$

TABLE 1. Radius ratio r/R.

Collector drop radius (μm)	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
300	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
200	0.87	0.96	0.98	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
150	0.77	0.93	0.97	0.97	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
100	0.50	0.79	0.91	0.95	0.95	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
70	0.20	0.58	0.75	0.84	0.88	0.90	0.92	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.97	1.0	1.02	1.04	2.3	4.0
60	0.05	0.43	0.64	0.77	0.84	0.87	0.89	0.90	0.91	0.91	0.91	0.91	0.91	0.92	0.93	0.95	1.0	1.03	1.7	3.0
50	0.005	0.40	0.60	0.70	0.78	0.83	0.86	0.88	0.90	0.90	0.90	0.90	0.89	0.88	0.88	0.89	0.92	1.01	1.3	2.3
40	0.001	0.07	0.28	0.50	0.62	0.68	0.74	0.78	0.80	0.80	0.80	0.78	0.77	0.76	0.77	0.77	0.78	0.79	0.95	1.4
30	0.0001	0.002	0.02	0.04	0.085	0.17	0.27	0.40	0.50	0.55	0.58	0.59	0.58	0.54	0.51	0.49	0.47	0.45	0.47	0.52
20	0.0001	0.0001	0.005	0.016	0.022	0.03	0.043	0.052	0.064	0.072	0.079	0.082	0.080	0.076	0.067	0.057	0.048	0.040	0.033	0.027
10	0.0001	0.0001	0.0001	0.014	0.017	0.019	0.022	0.027	0.030	0.033	0.035	0.037	0.038	0.038	0.037	0.036	0.035	0.032	0.029	0.027

Hall (*J. Atmos. Sci.* 1980)
(compilation of many theoretical studies and laboratory measurements)

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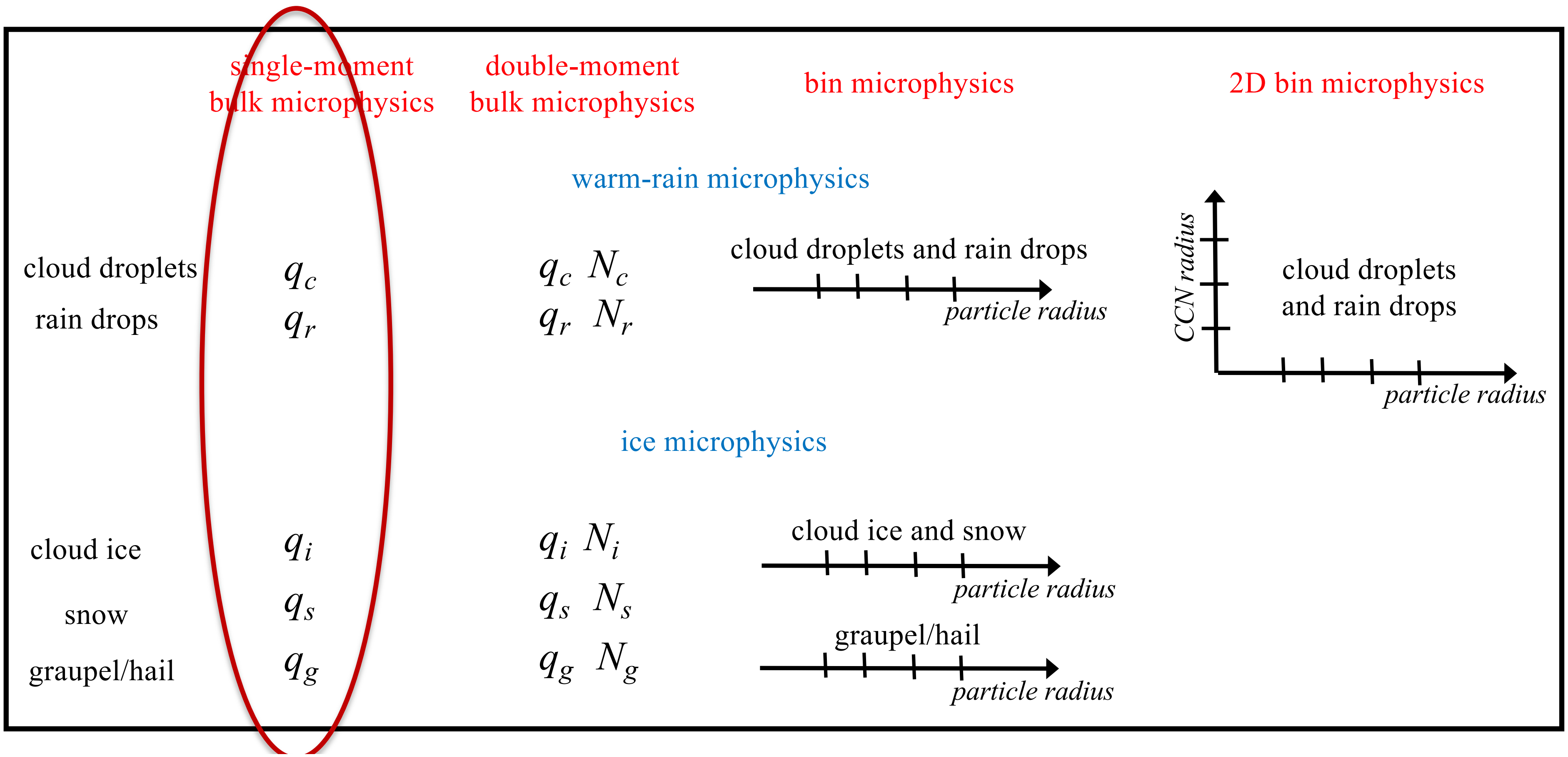

BULK RAIN/ICE MODEL
(Lin et al. 1983, Rutledge and Hobbs 1984)

$$\frac{\partial \rho_o \theta}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} \theta) = \frac{L_v \theta_e}{c_p T_e} S_1 + \frac{L_s \theta_e}{c_p T_e} S_2 + \frac{L_f \theta_e}{c_p T_e} S_3 + D_\theta$$

Water vapor

$$\frac{\partial \rho_o q_v}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_v) = S_4 + D_{q_v}$$

Cloud water

$$\frac{\partial \rho_o q_c}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_c) = S_5 + D_{q_c}$$

Cloud ice

$$\frac{\partial \rho_o q_i}{\partial t} + \nabla \cdot (\rho_o \mathbf{u} q_i) = S_6 + D_{q_i}$$

$$\frac{\partial \rho_o q_r}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^r \mathbf{k}) q_r] = S_7 + D_{q_r}$$

Rain

$$\frac{\partial \rho_o q_s}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^s \mathbf{k}) q_s] = S_8 + D_{q_s}$$

Snow

$$\frac{\partial \rho_o q_g}{\partial t} + \nabla \cdot [\rho_o (\mathbf{u} - V_T^g \mathbf{k}) q_g] = S_9 + D_{q_g}$$

Traditional approach to bulk cloud microphysics

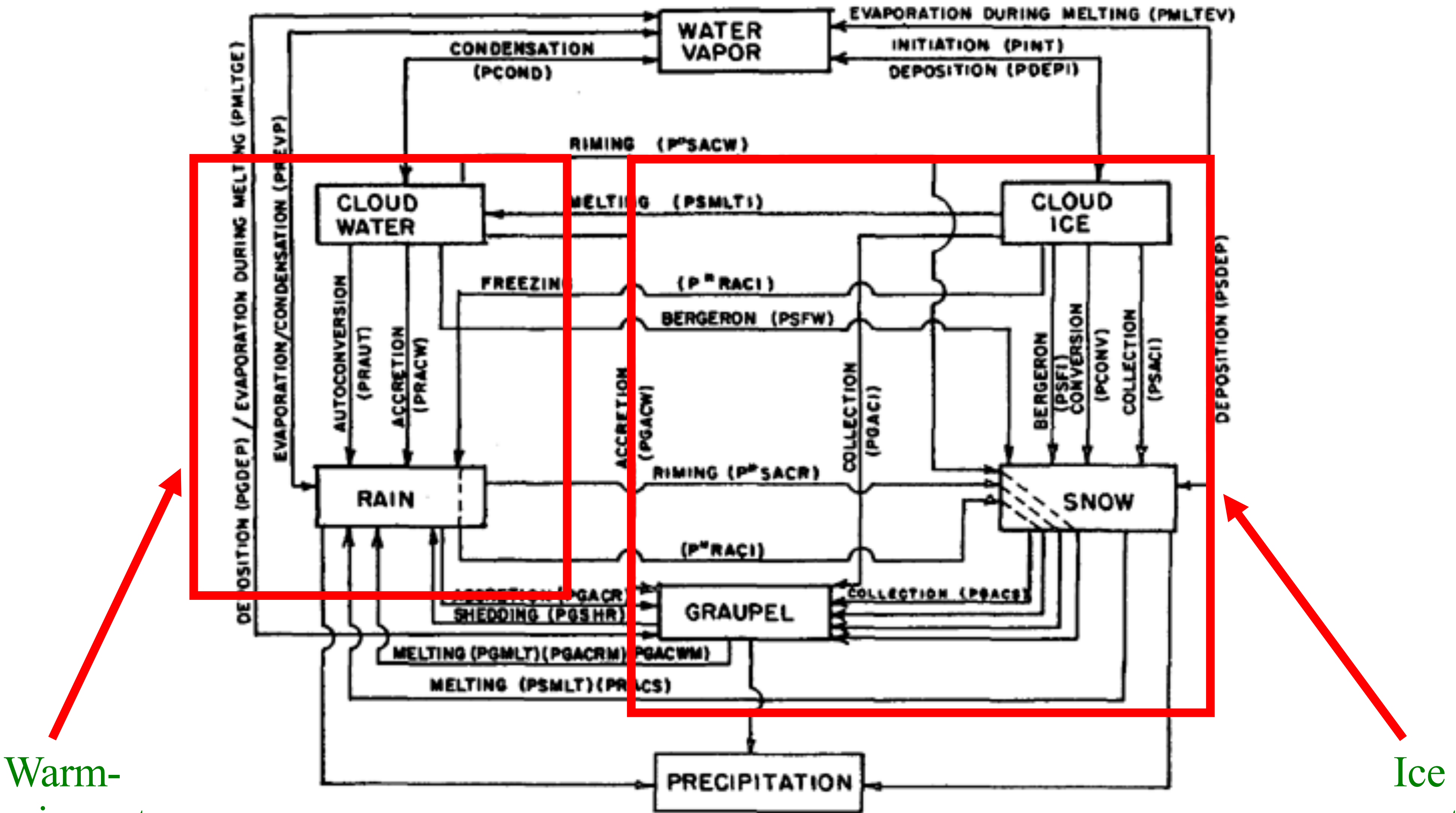

FIG. 1. Schematic depicting the cloud and precipitation processes included in the model for the study of narrow cold-frontal rainbands.

Lin et al. JCAM 1983, Rutledge and Hobbs JAS 1984

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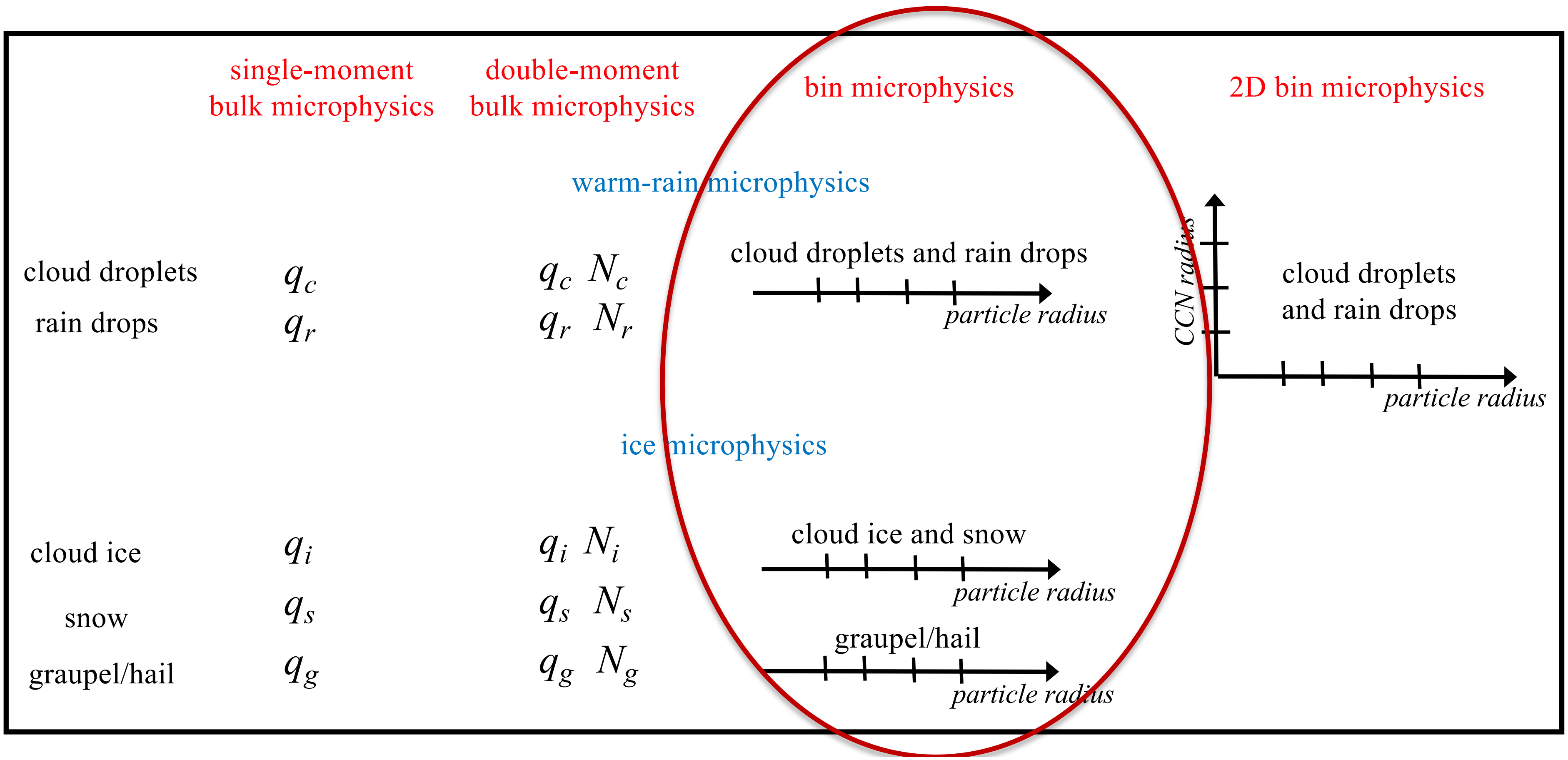

Is bin microphysics the ultimate scheme?

Such a scheme is often used as a benchmark for bulk schemes (e.g., deriving formulas for bulk schemes)…

But, there are issues:

- physical complexity, especially for ice;
- stochastic („lucky droplets“) vs deterministic (Smoluchowski eq.) rain onset?
- numerical aspects;
- how to account for subgrid-scale processes?

Idealized Simulations of a Squall Line from the MC3E Field Campaign Applying Three Bin Microphysics Schemes: Dynamic and Thermodynamic Structure

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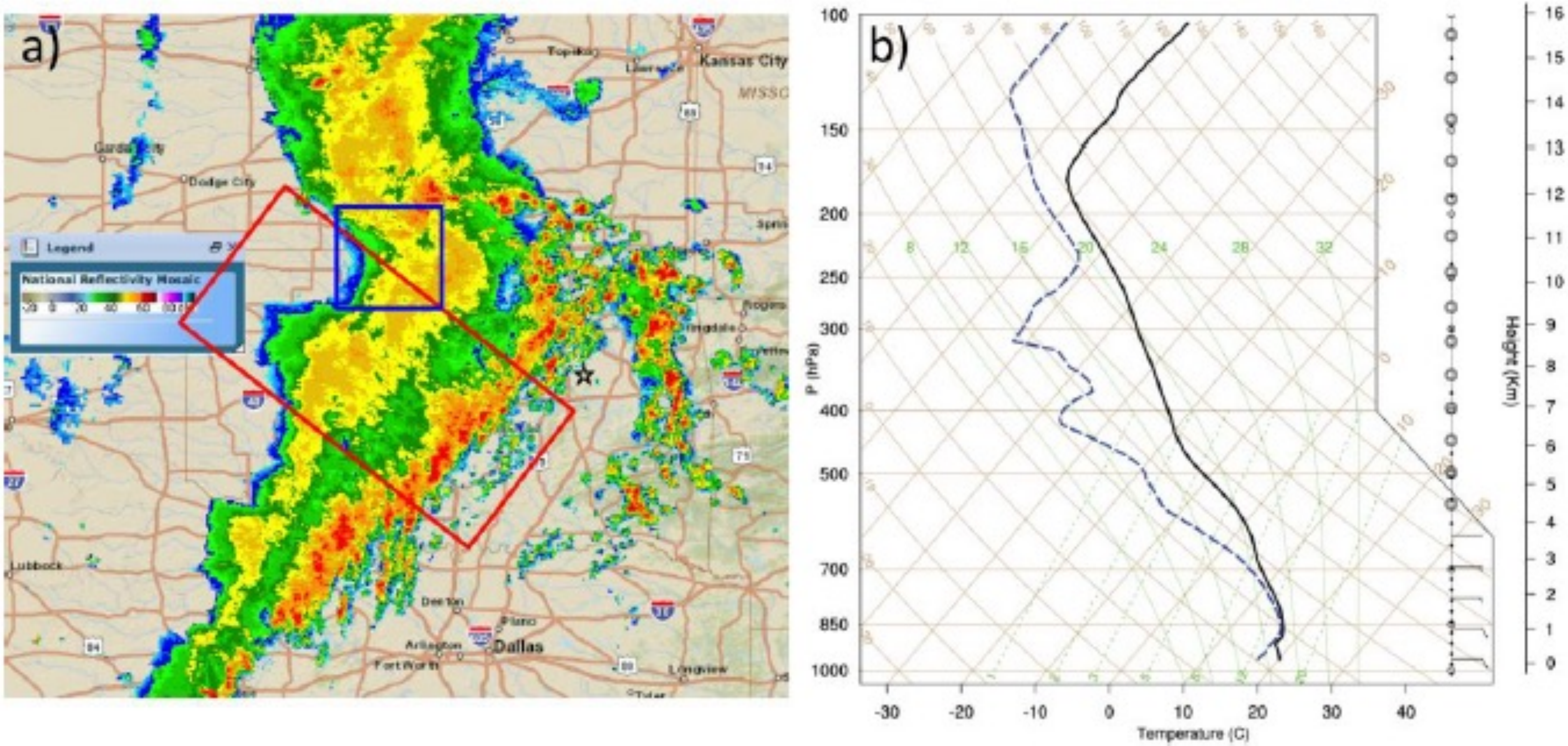
WOJCIECH W. GRABOWSKI,[a] XIA CHU,[c] ISTVÁN GERESDI,[e] KIRK NORTH,[f] RONALD STENZ,[g]
YANG GAO,[b] XIAOFENG LOU,[h] AARON BANSEMER,[a] ANDREW J. HEYMSFIELD,[a]
GREG M. MCFARQUHAR,[d,a] AND ROY M. RASMUSSEN[a]

Mon. Wea. Rev. 2017



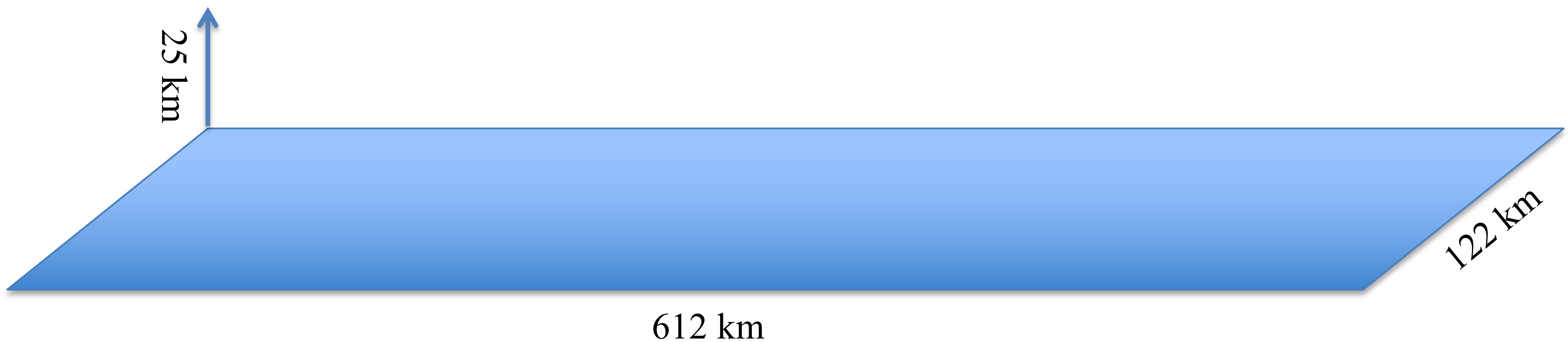
Setup of WRF bin simulations:

bowling alley horizontal domain; open at the ends, periodic across

1 km/0.25 km horizontal/vertical grid length, 3 sec time step

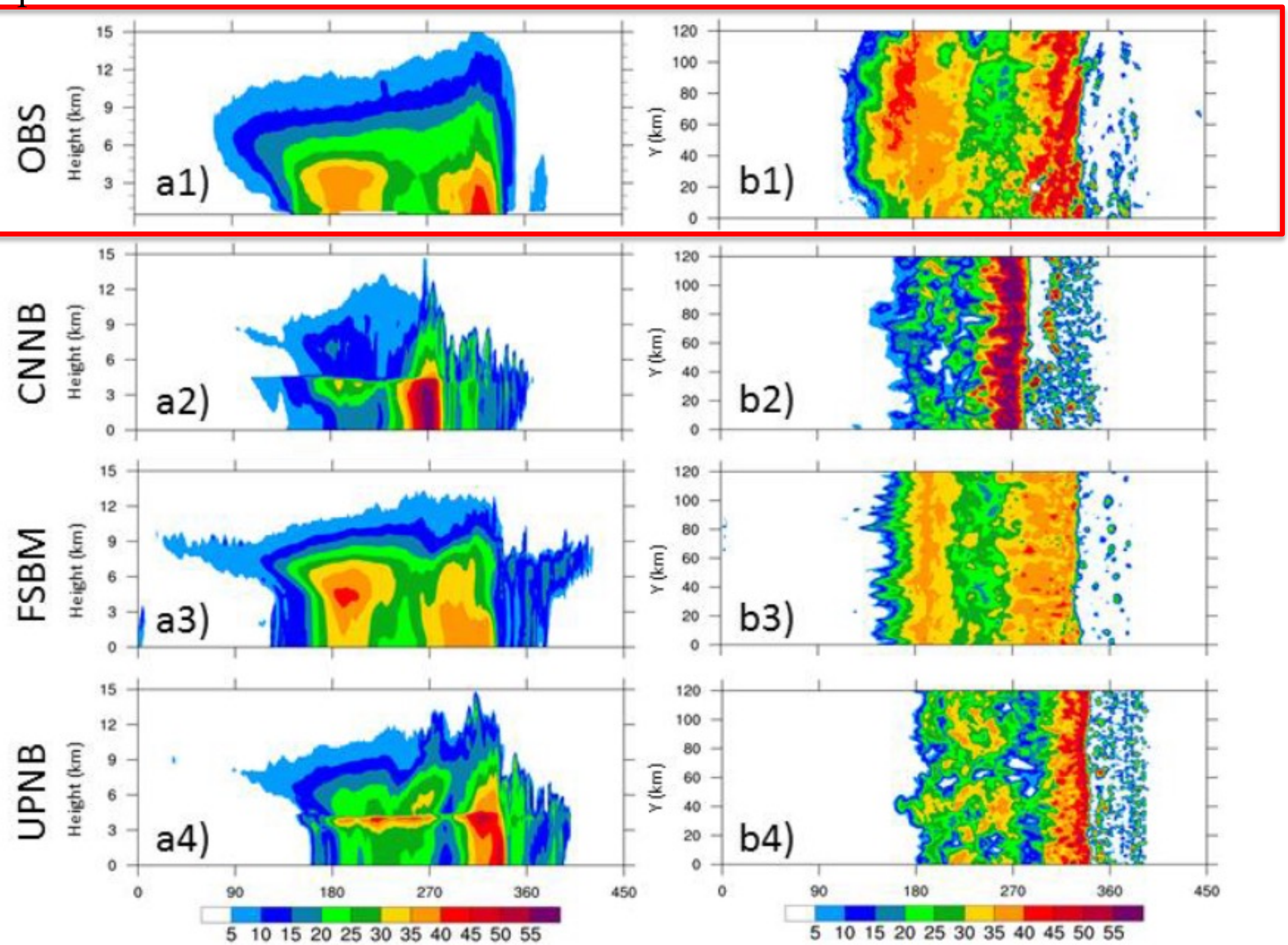
single sounding (from observed environment ahead of the squall line) initialization, low-level convergence applied to initiate convection

model run for 6 hours, two initial hours considered as spinup



Radar reflectivity

composite of observations



2-moment, Z. Lebo et al. (Caltech, NCAR)

1-moment, A. Khain et al. (Hebrew U.)

1-moment, I. Geresdi et al. (U. of Pecs)

line average

horizontal cross section at 2 km

Can we do better?

Lagrangian treatment of the condensed phase: "Lagrangian Cloud Model", "Super-droplet method":

The super-droplet method for the numerical simulation of clouds and precipitation: A particle-based and probabilistic microphysics model coupled with a non-hydrostatic model

S. Shima,[a]* K. Kusano,[c] A. Kawano,[a] T. Sugiyama[a] and S. Kawahara[b]

Cloud-aerosol interactions for boundary layer stratocumulus in the Lagrangian Cloud Model

M. Andrejczuk,[1] W. W. Grabowski,[2] J. Reisner,[3] and A. Gadian[1]

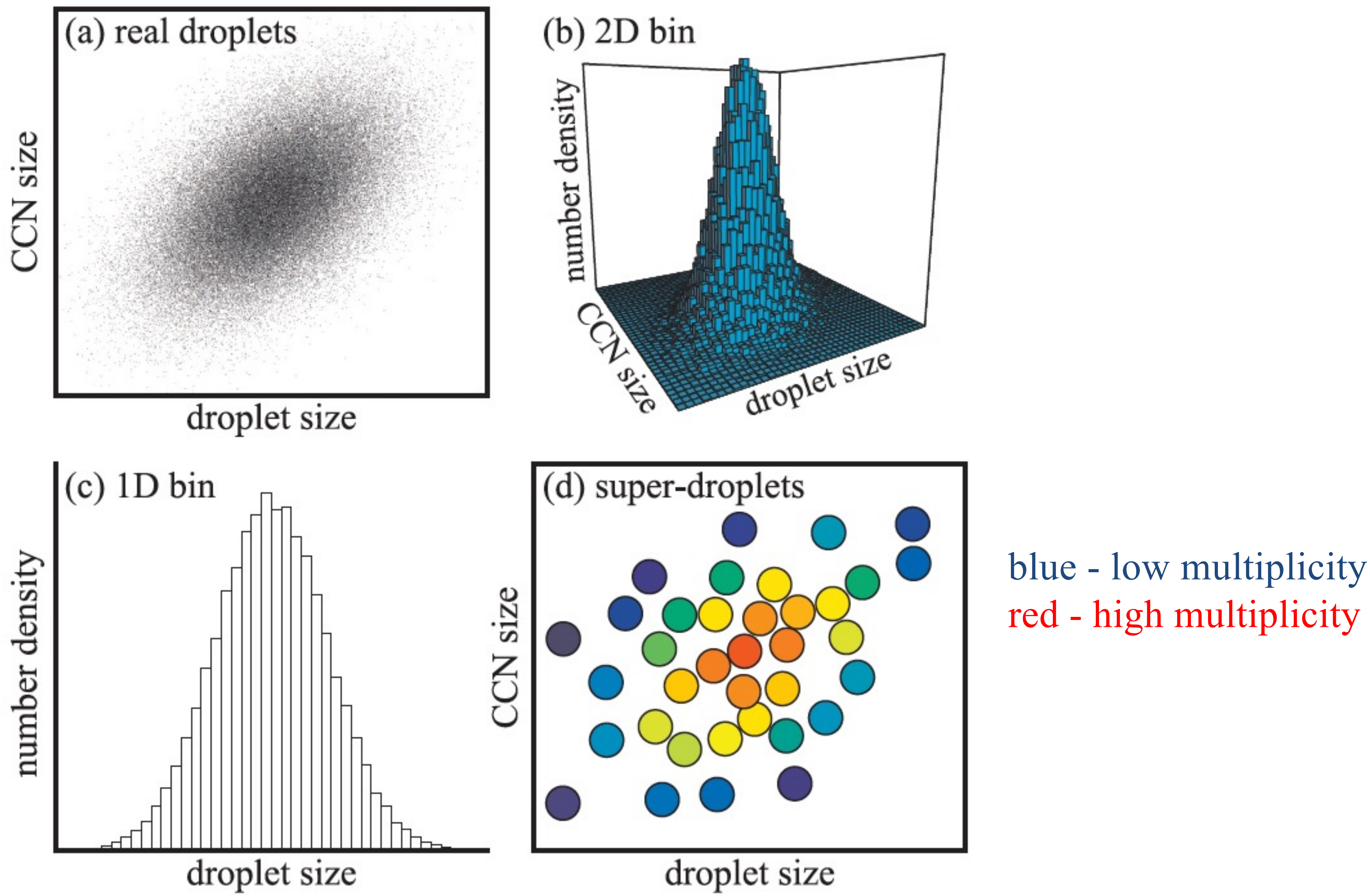
libcloudph++ 1.0: a single-moment bulk, double-moment bulk, and particle-based warm-rain microphysics library in C++

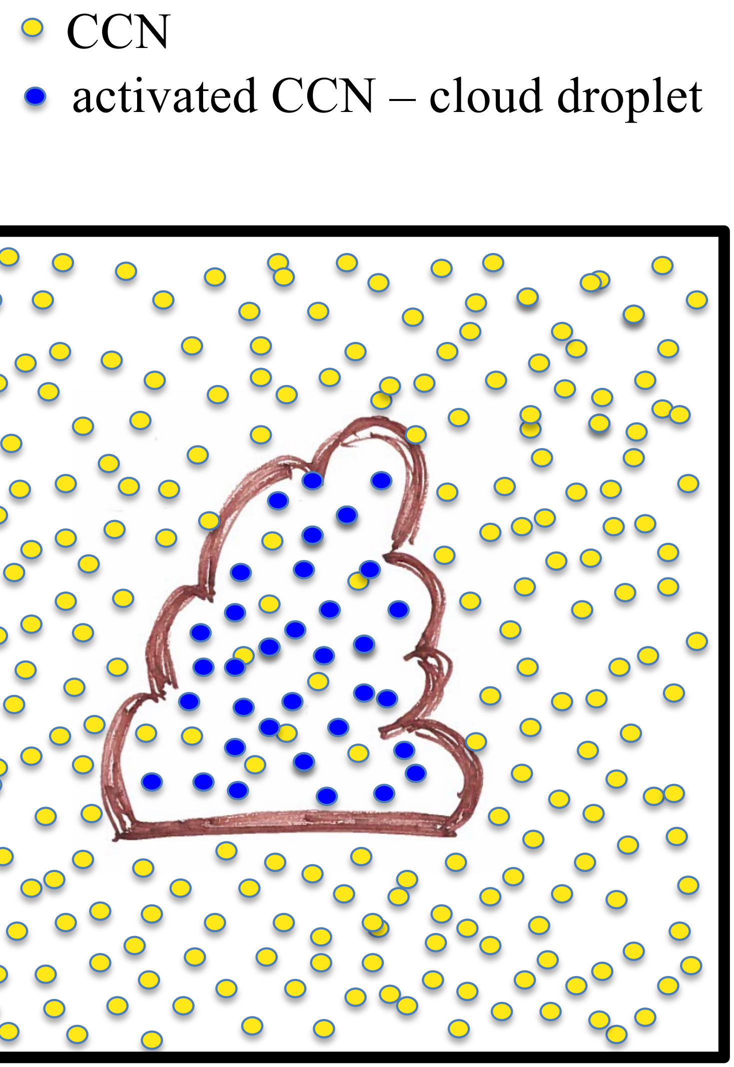
S. Arabas[1], A. Jaruga[1], H. Pawlowska[1], and W. W. Grabowski[2]

[1]Institute of Geophysics, Faculty of Physics, University of Warsaw, Warsaw, Poland
[2]National Center for Atmospheric Research (NCAR), Boulder, CO, USA

A new method for large-eddy simulations of clouds with Lagrangian droplets including the effects of turbulent collision

T Riechelmann[1,3], Y Noh[2] and S Raasch[1]


(a) real droplets
CCN size
droplet size
(b) 2D bin
number density
CCN size
droplet size
(c) 1D bin
number density
droplet size
(d) super-droplets
CCN size
droplet size
blue - low multiplicity
red - high multiplicity

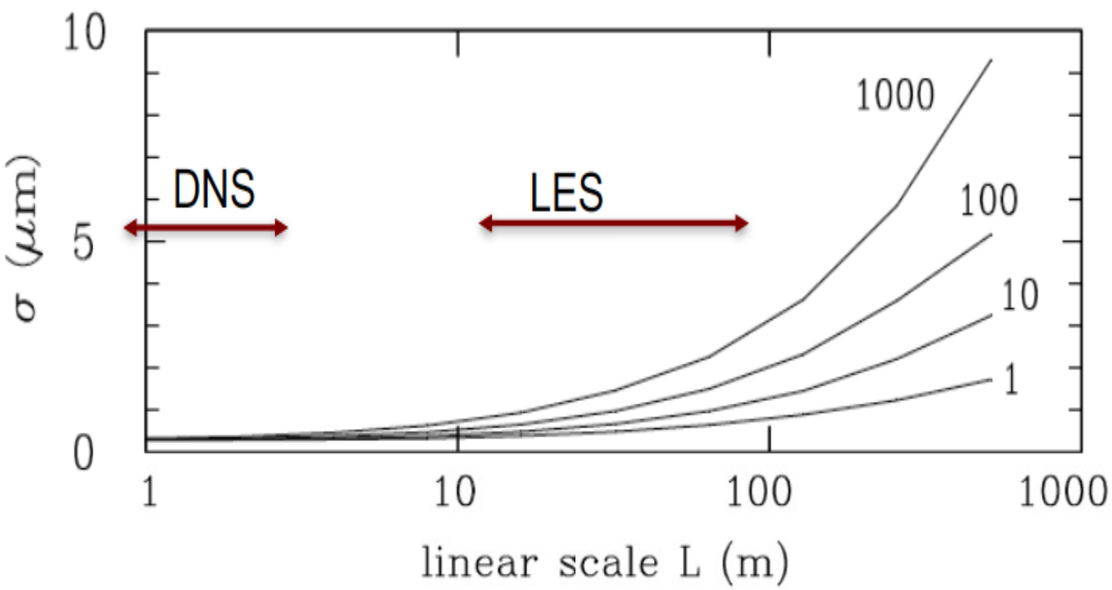


Lagrangian warm-rain microphysics
(e.g., Andrejczuk et al., Shima et al., Arabas et al. and others)

Summary:

Lagrangian approach to model cloud processes provides a straightforward methodology when compared to existing Eulerian bin microphysics schemes.

Since typical grid lengths in cloud simulations are a few 10s of meters, the impact of subgrid-scale processes on the droplet spectrum needs to be included. This is straightforward when the Lagrangian approach is used, but difficult (impossible?) for traditional Eulerian LES cloud models.



Grabowski and Abade *JAS* 2017

Extension of the Lagrangian approach to include ice processes seems straightforward and is pursued by several groups (Japan, Germany, Poland). Application to deep convection simulation will become a reality soon…