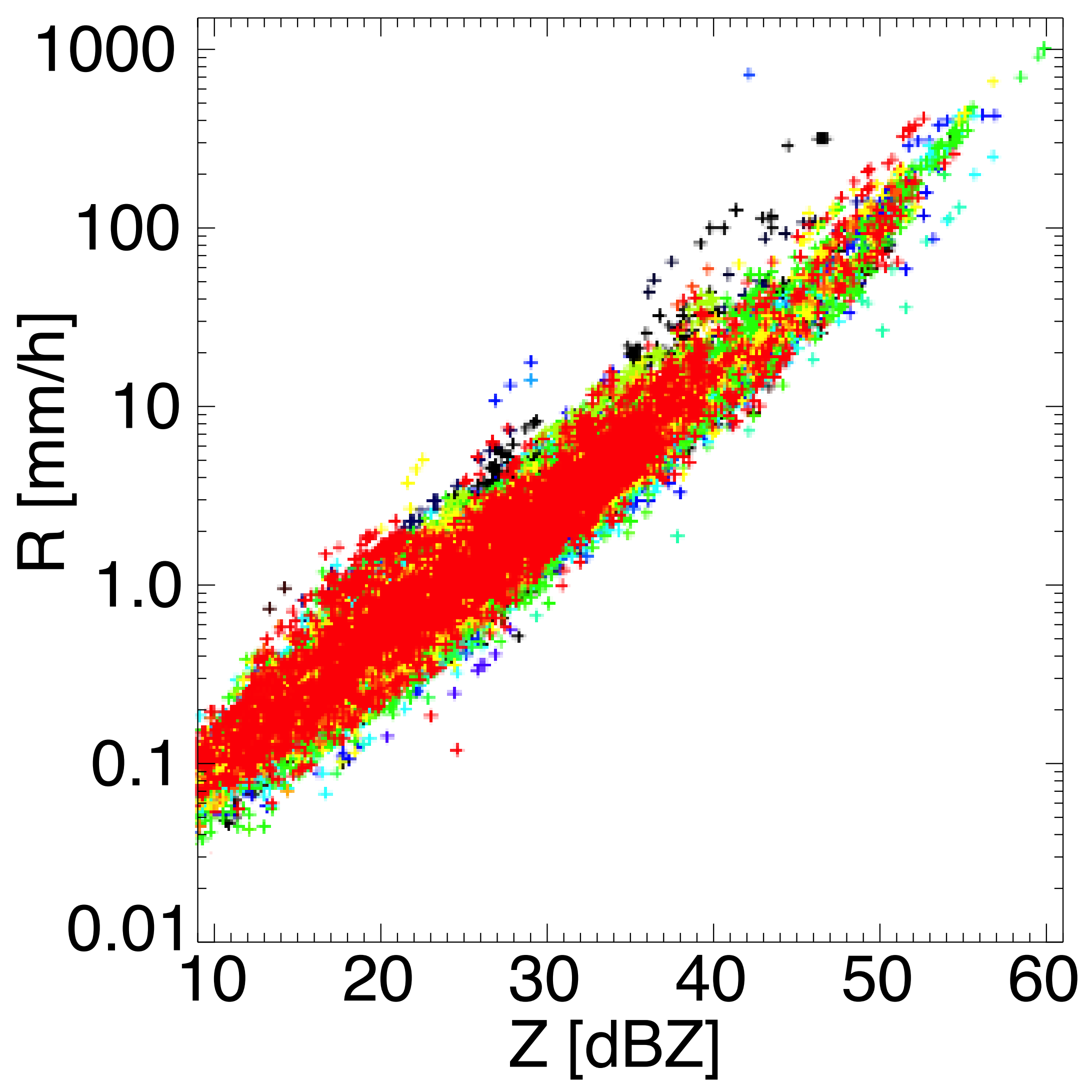
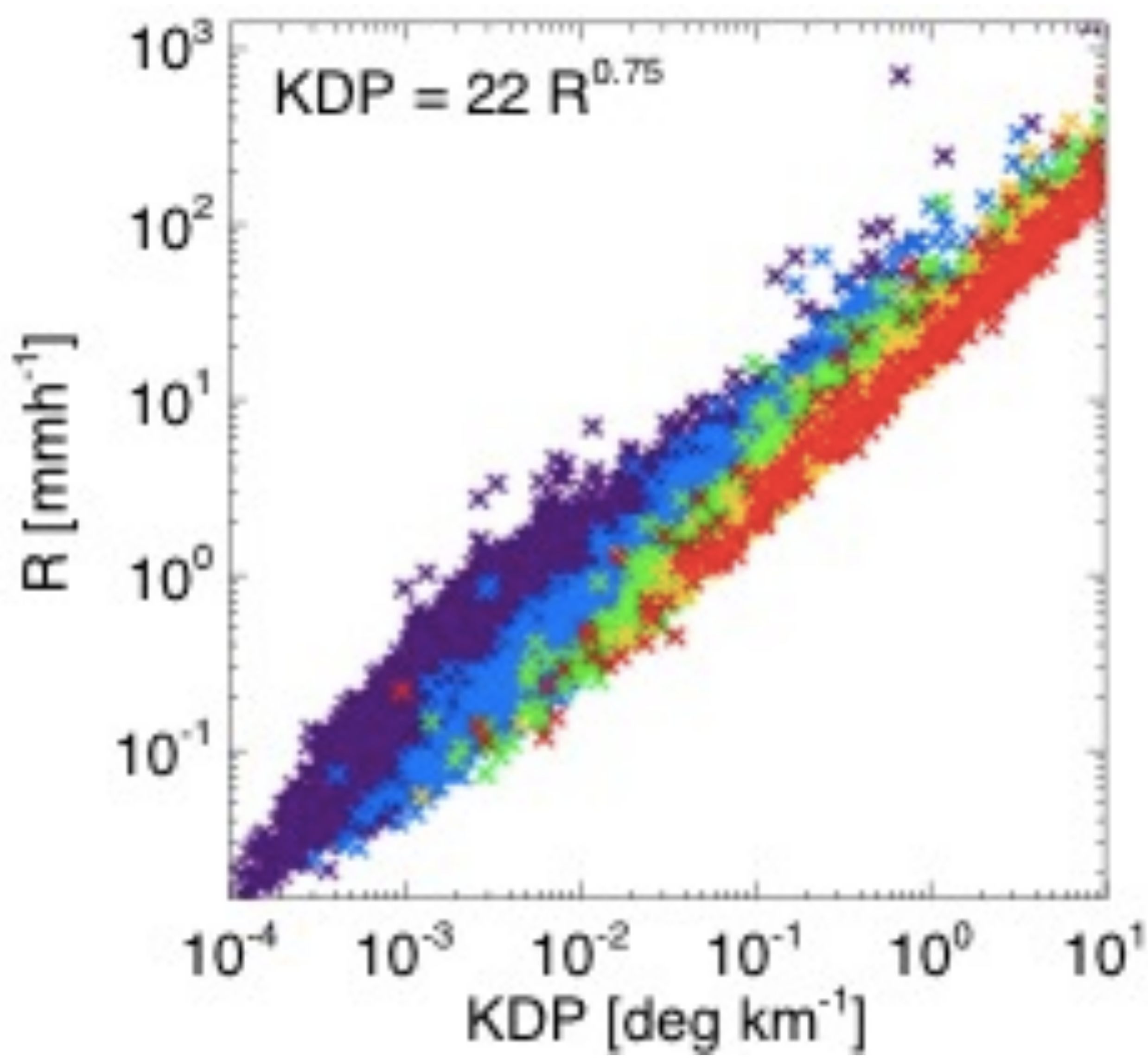
QPE “Techniques”

Multivariate relationships

Colour indicates values of ZDR; it is clear that ZDR adds information to the KDP-R relationship





Algorithms for rainfall estimation

$$R(Z) = 1.7010^{-2} Z^{0.714}$$

$$R^{(1)}(Z, Z_{DR}) = 1.4210^{-2} Z^{0.77} Z_{dr}^{-1.67}$$

Works well for continental rain

$$R^{(2)}(Z, Z_{DR}) = 6.7010^{-3} Z^{0.93} Z_{dr}^{-3.43}$$

Works well for tropical rain

$$R(Z, K_{DP}) = \begin{matrix} R(Z)\ if\ \ Z < 40\,dBZ \\ R(K_{DP})\ if\ \ Z > 40\,dBZ \end{matrix}$$

Works well for all rain types

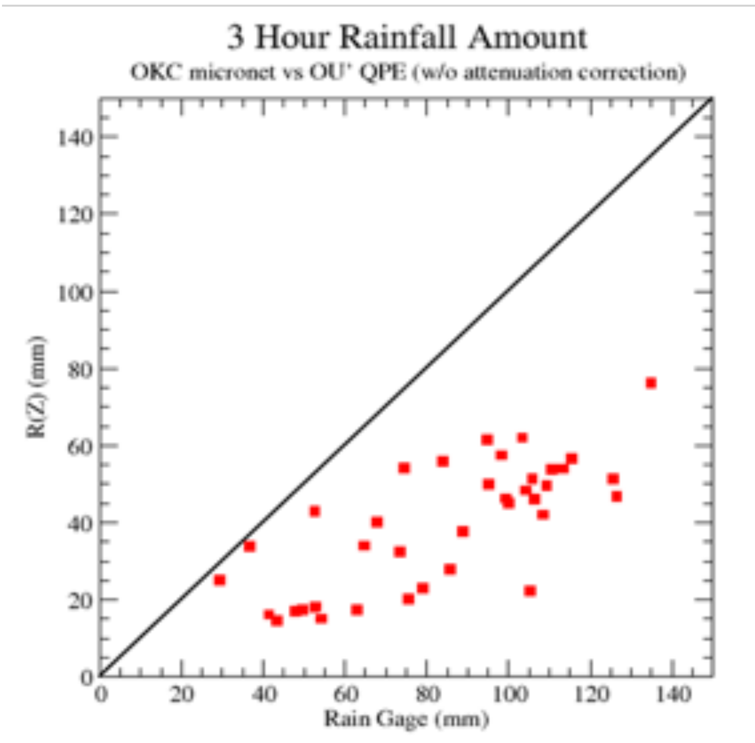
$$R(K_{DP}) = 44.0 |\, K_{DP}\, |^{0.822}\ sign(K_{DP})$$

$$R(Z, Z_{DR}, K_{DP}) = \begin{matrix} R^{(1)}(Z, Z_{DR})\ if\ \ Z < 40\,dBZ \\ R(K_{DP})\ if\ \ Z > 40\,dBZ \end{matrix}$$

Courtesy of Alexander Ryzhkov

3 hour rain total estimates at C band

R(Z) before correction



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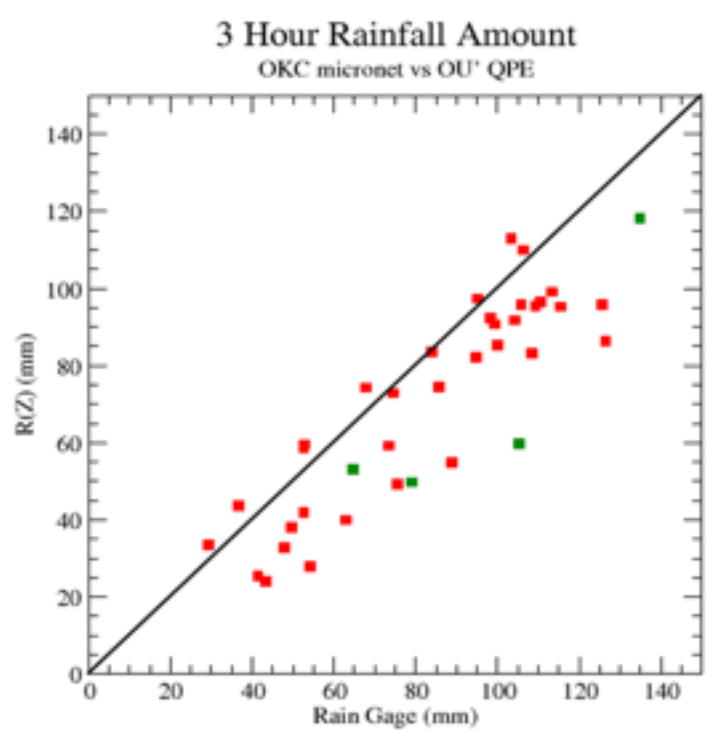
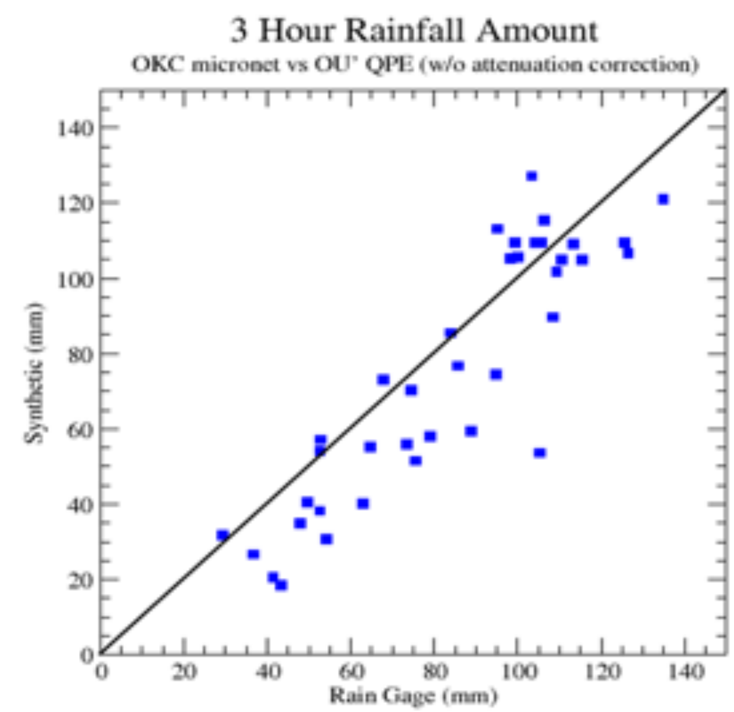
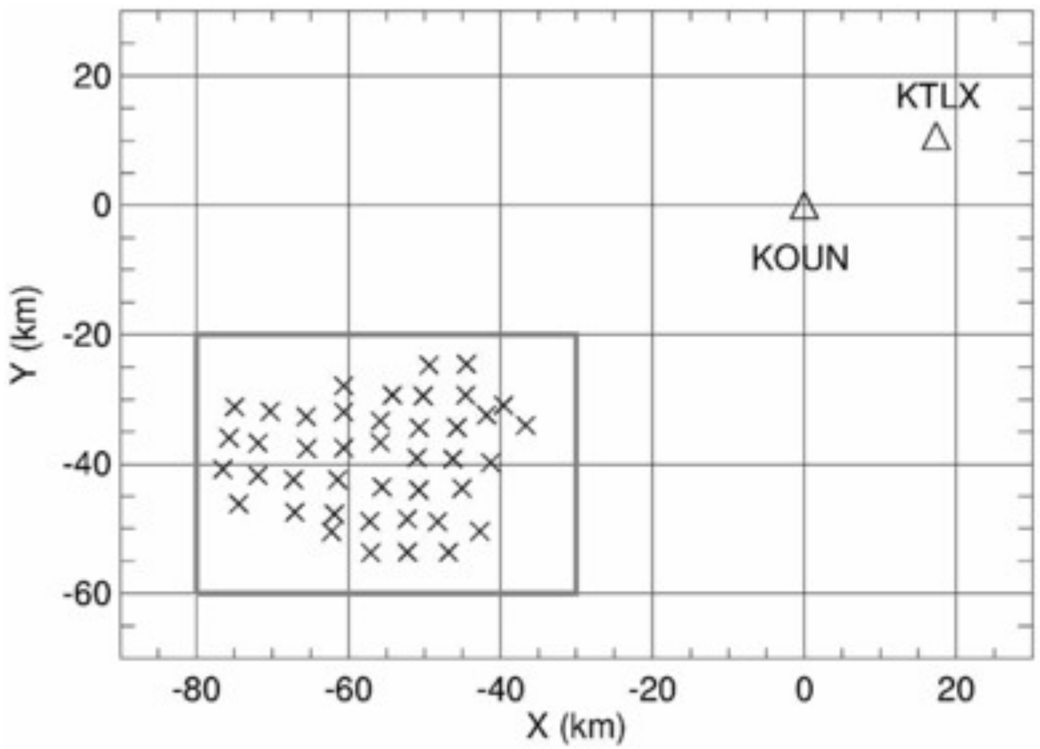


$R(Z, K_{DP})$



Courtesy of Alexander Ryzhkov

Comparison with gages of 1-h accumulations



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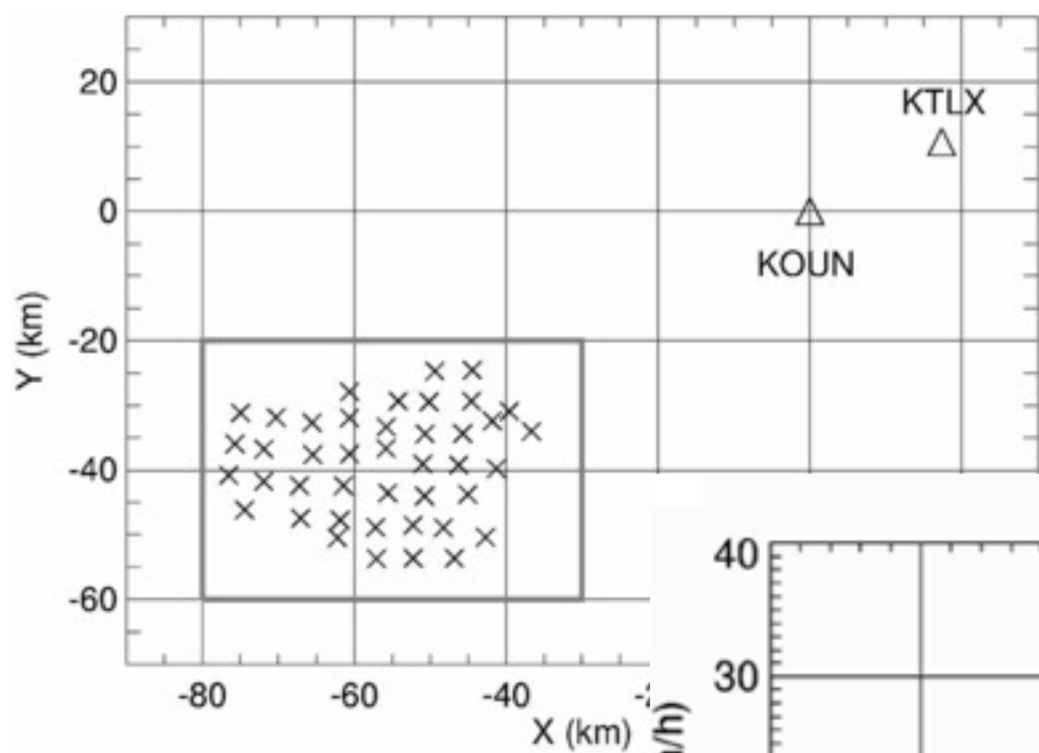
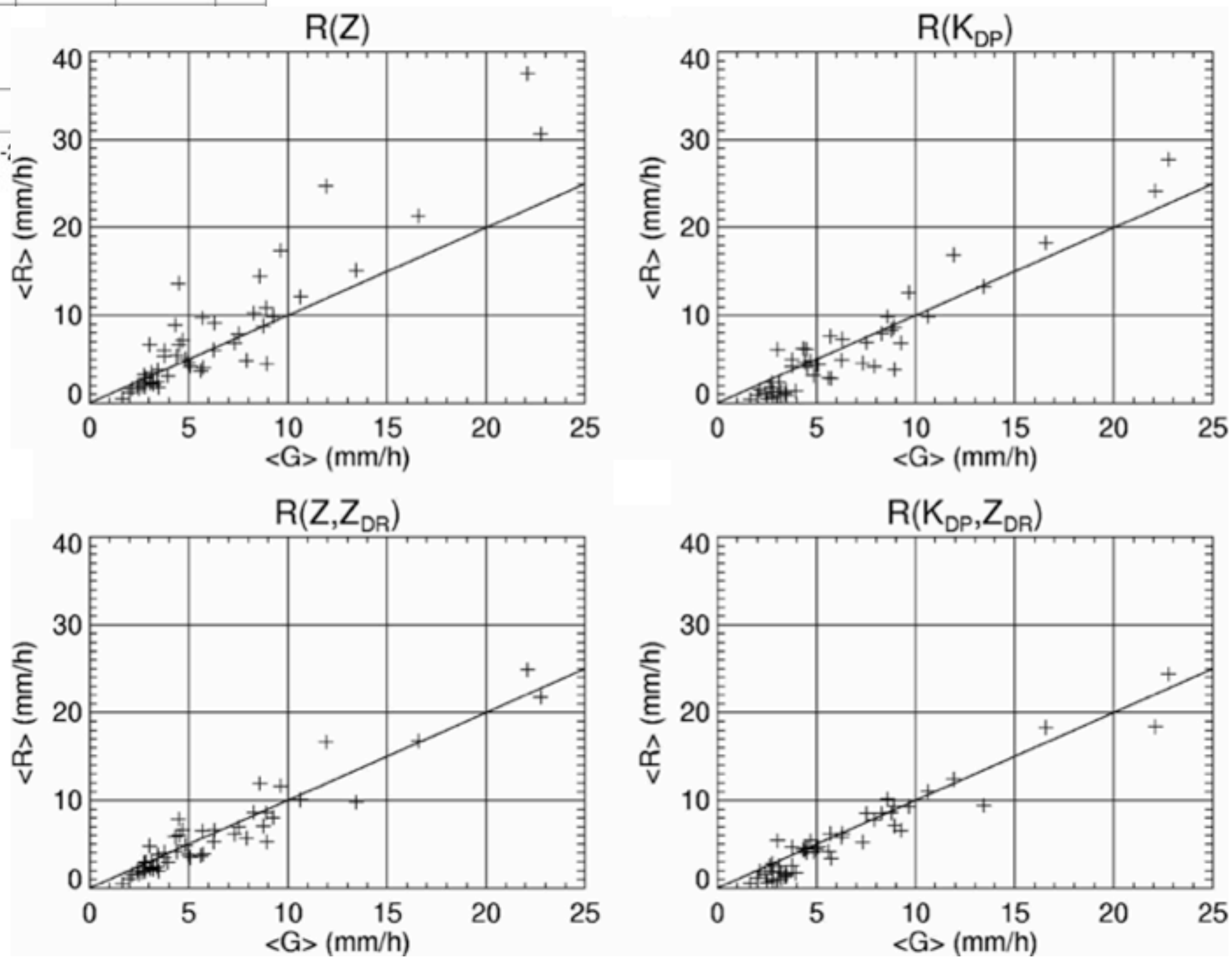
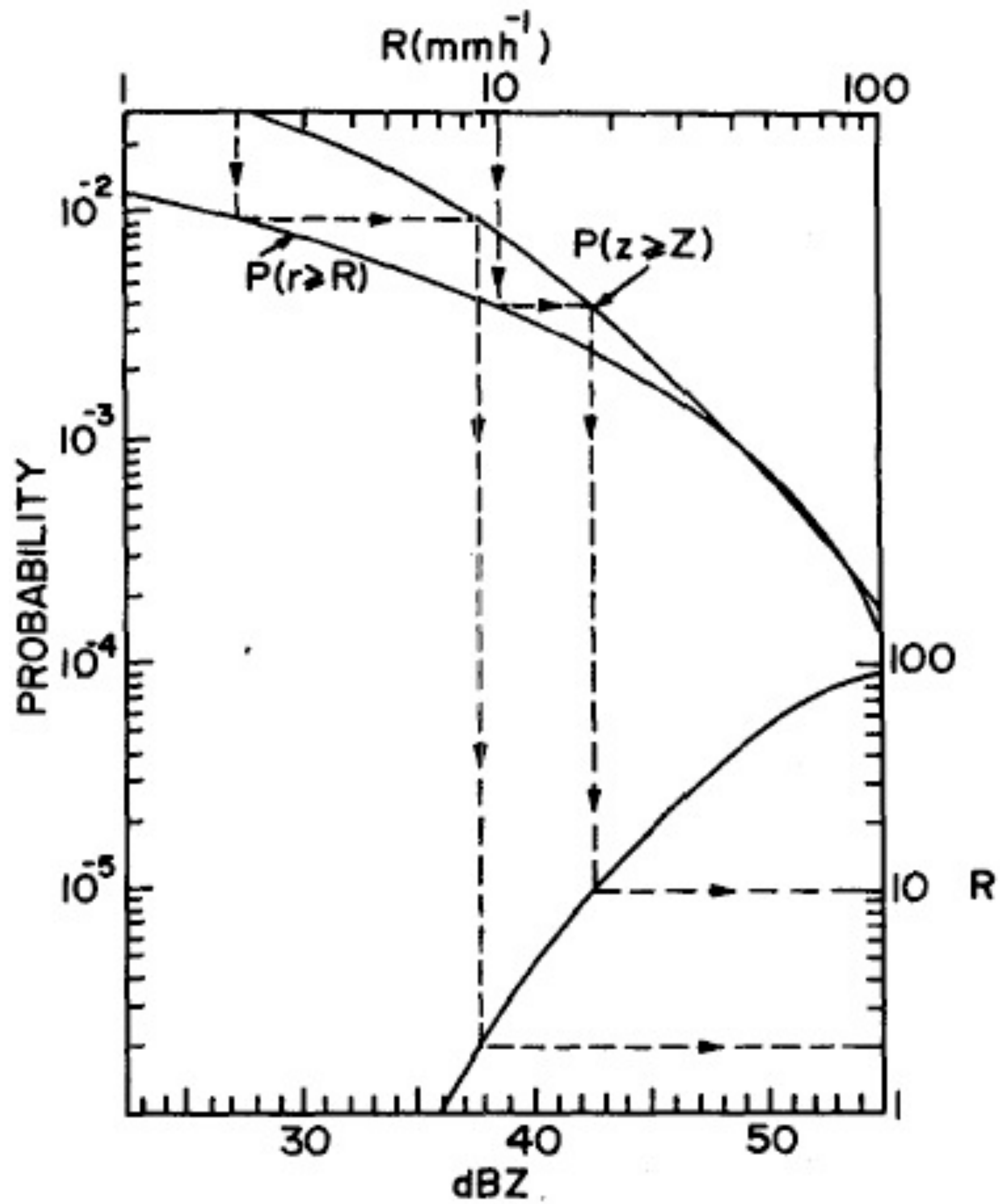




Courtesy of Alexander Ryzhkov

A shortcut: probability matching



Calheiros and Zawadzki, 1987, JAM

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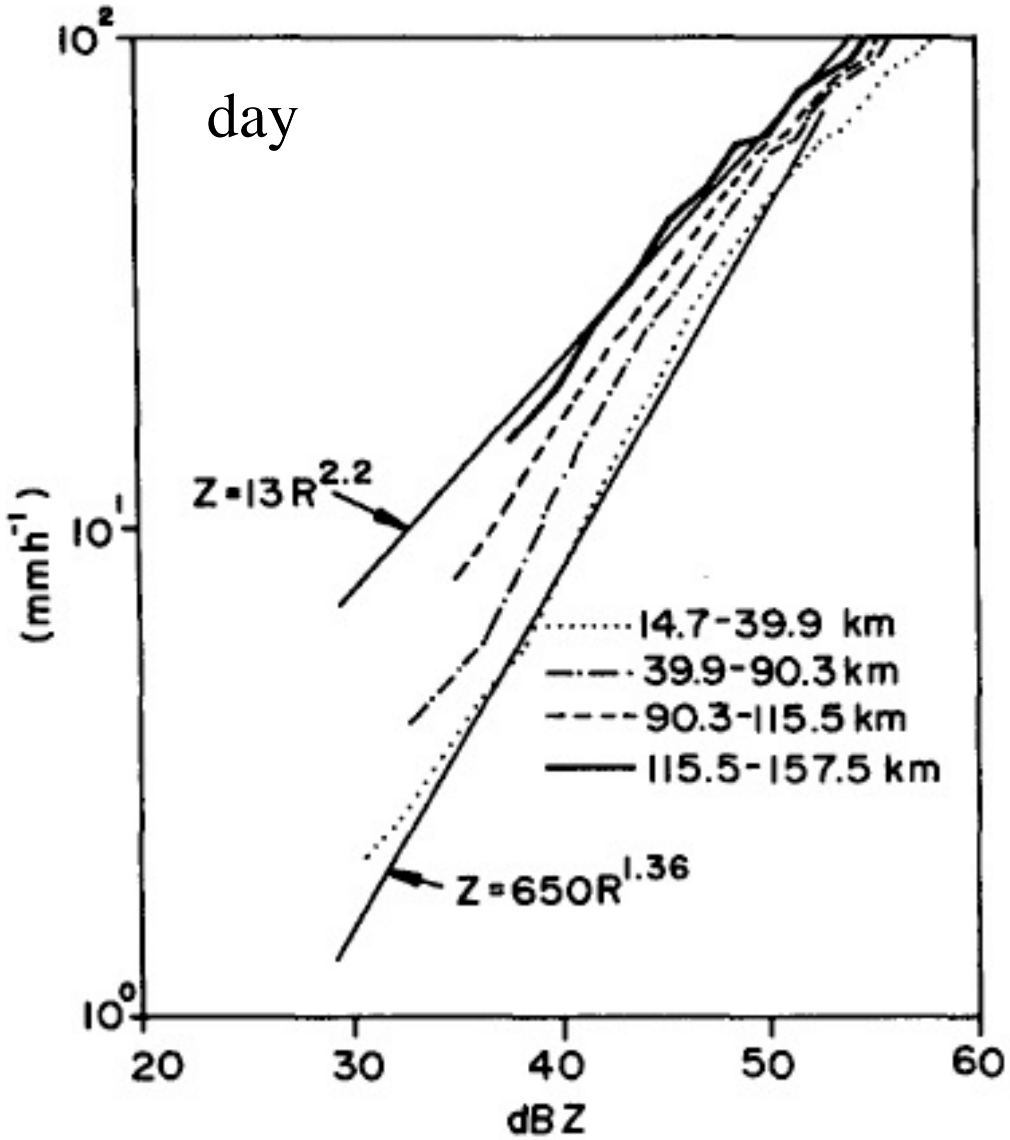
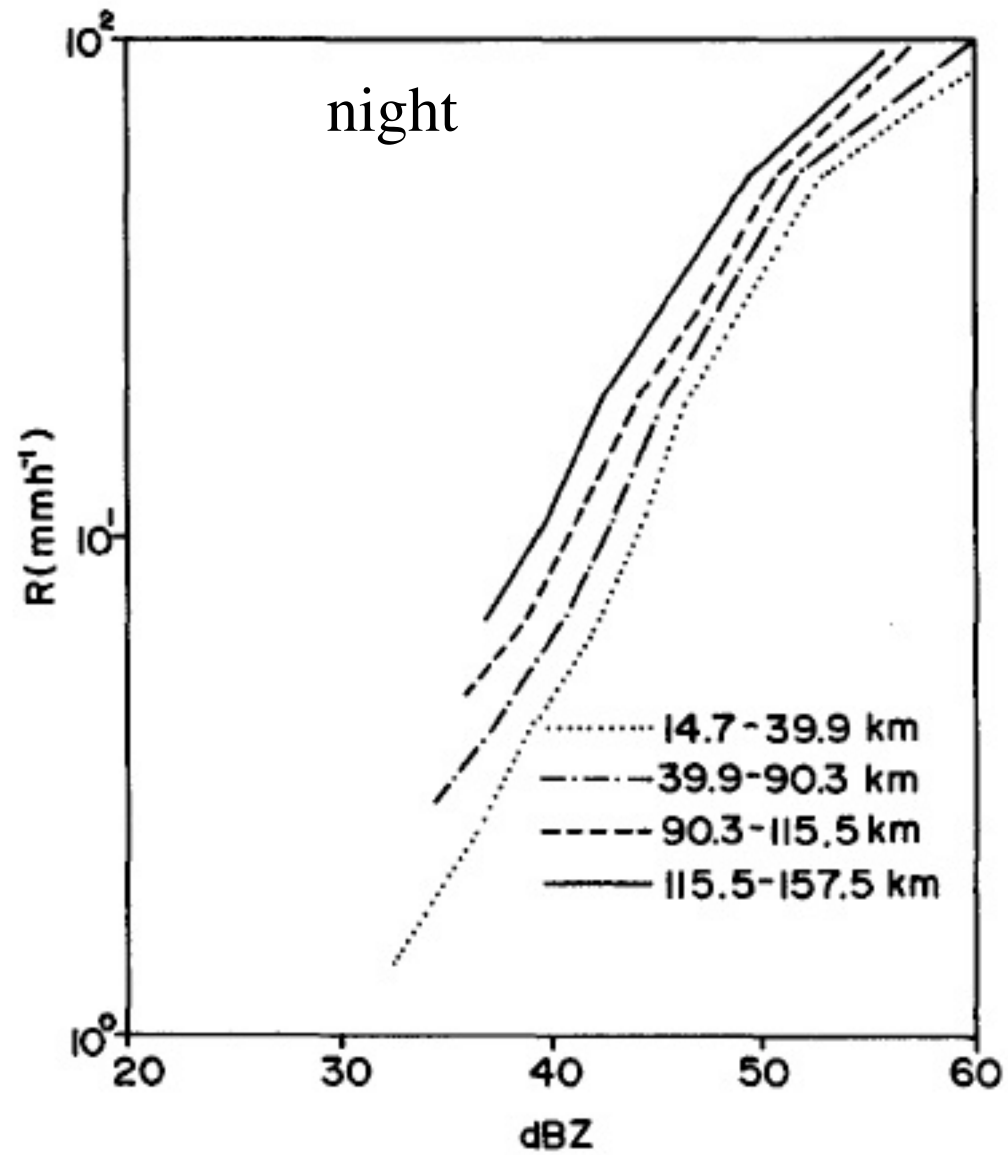
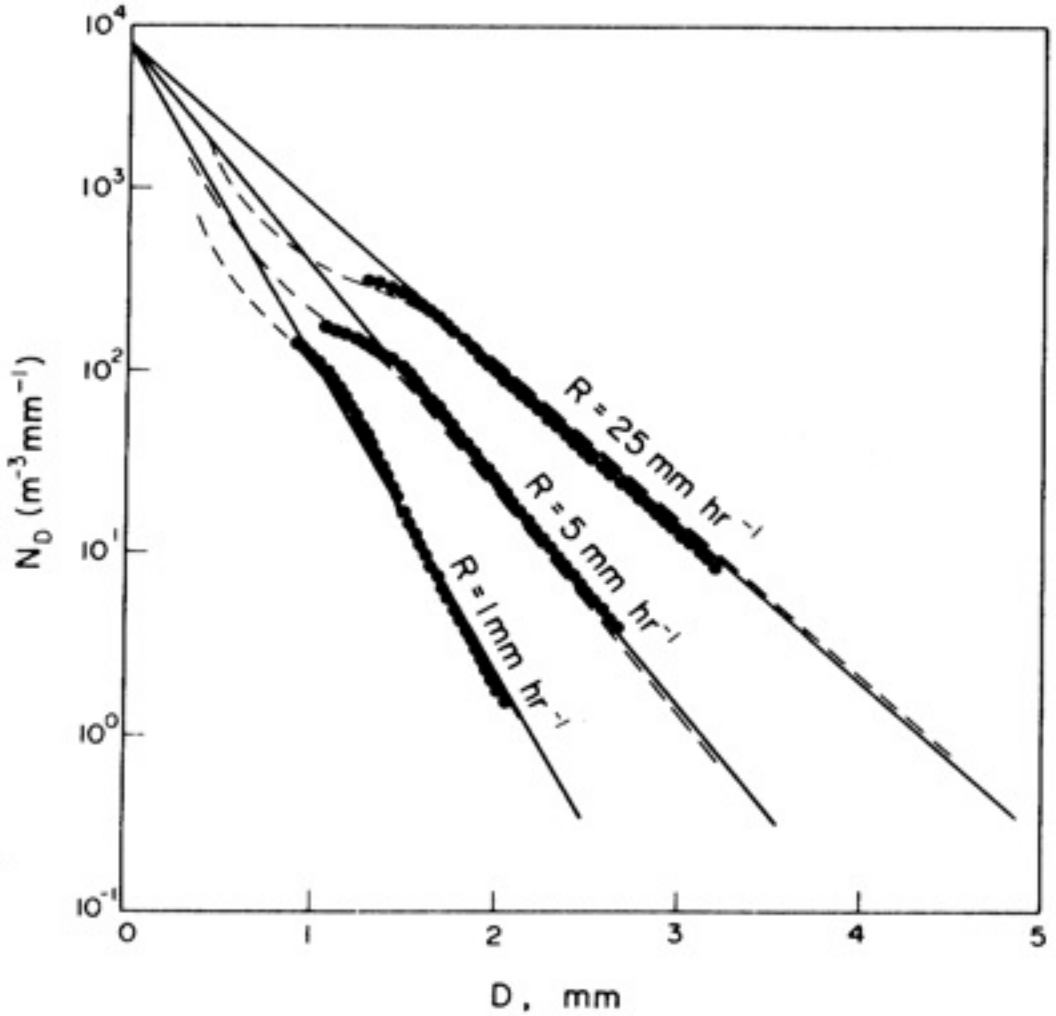




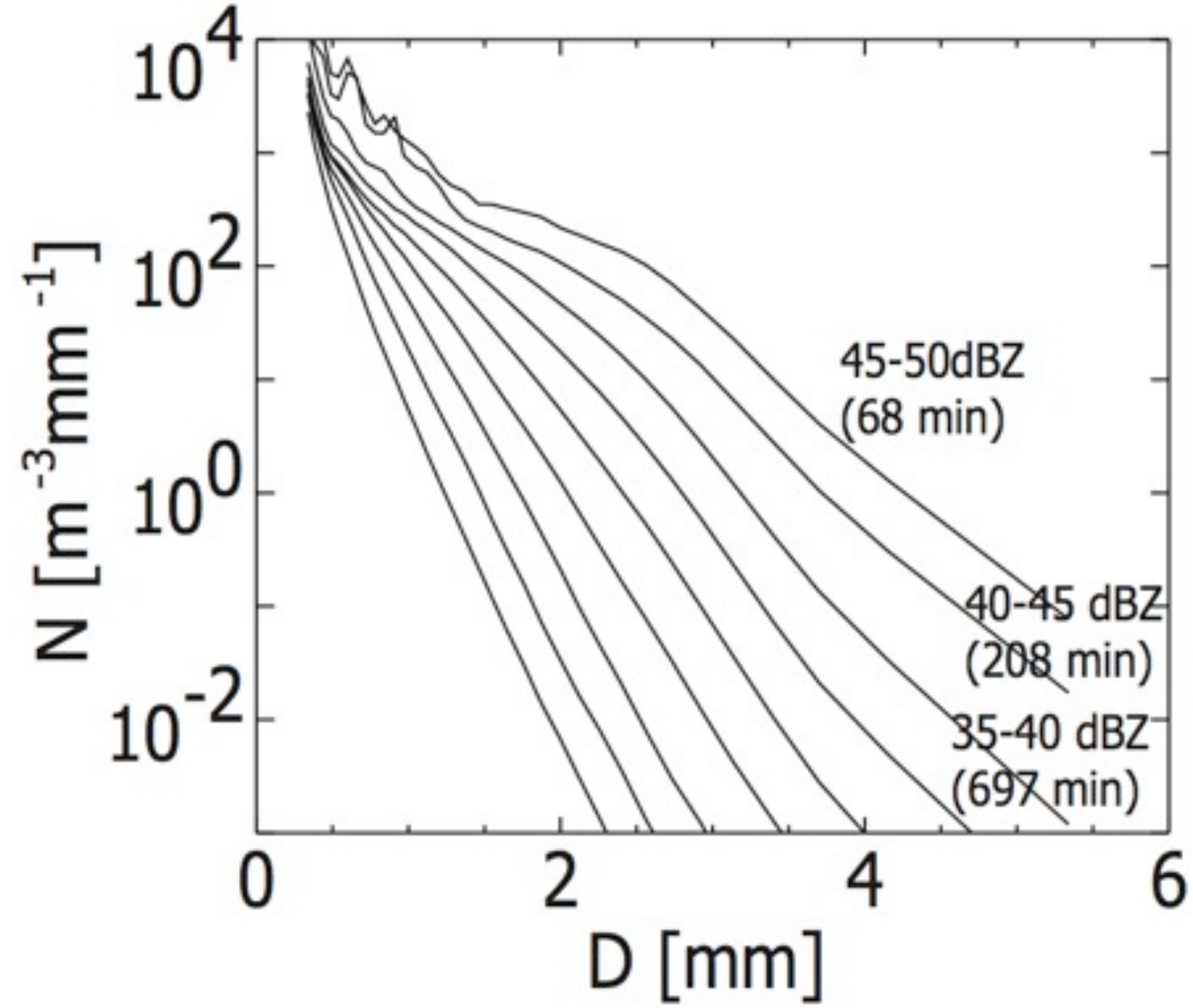
Calheiros and Zawadzki, 1987, JAM

Why do we use power-laws to relate the different moments of the DSDs?

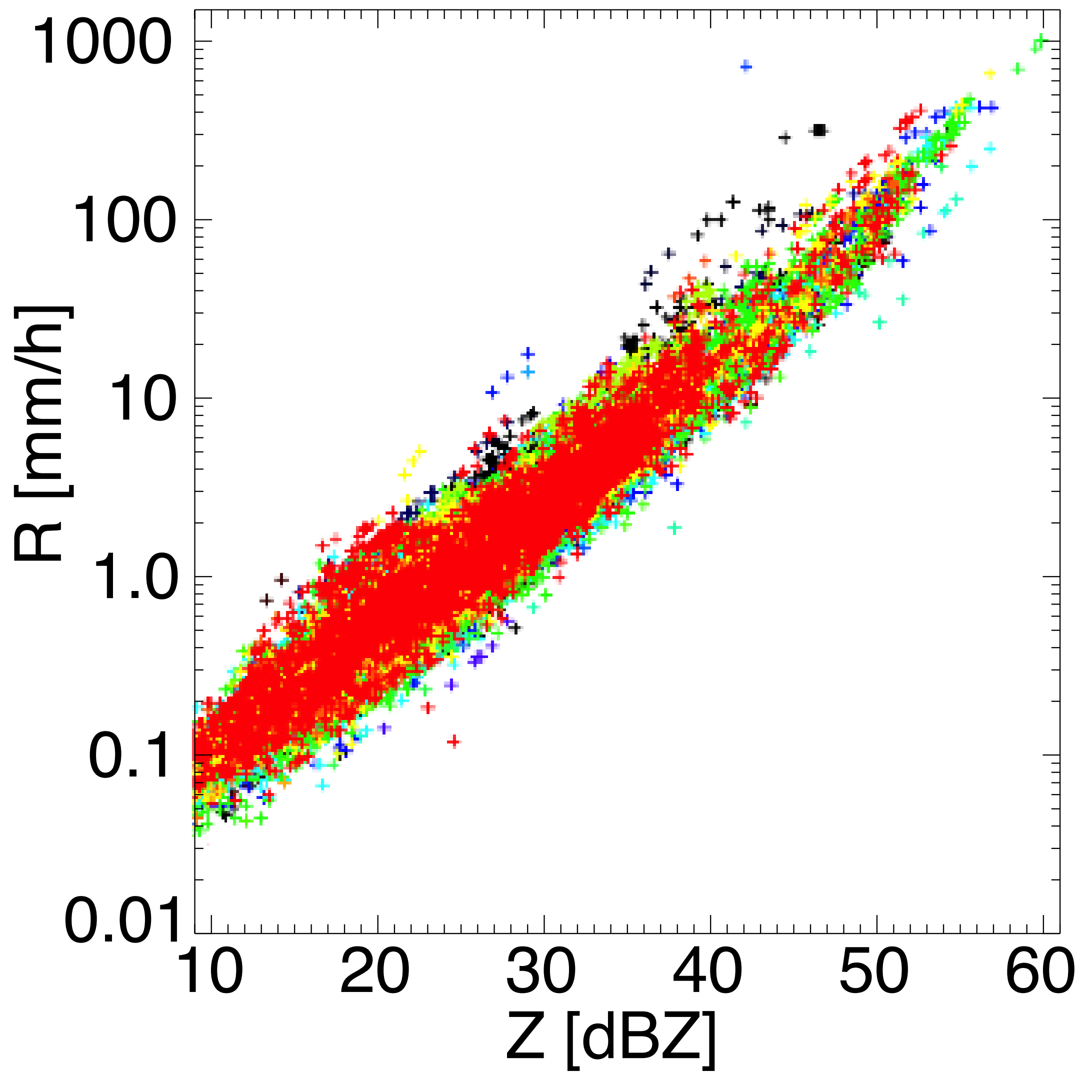
Compatible with a power-law Z-R



Not compatible with a power-law



Climatology



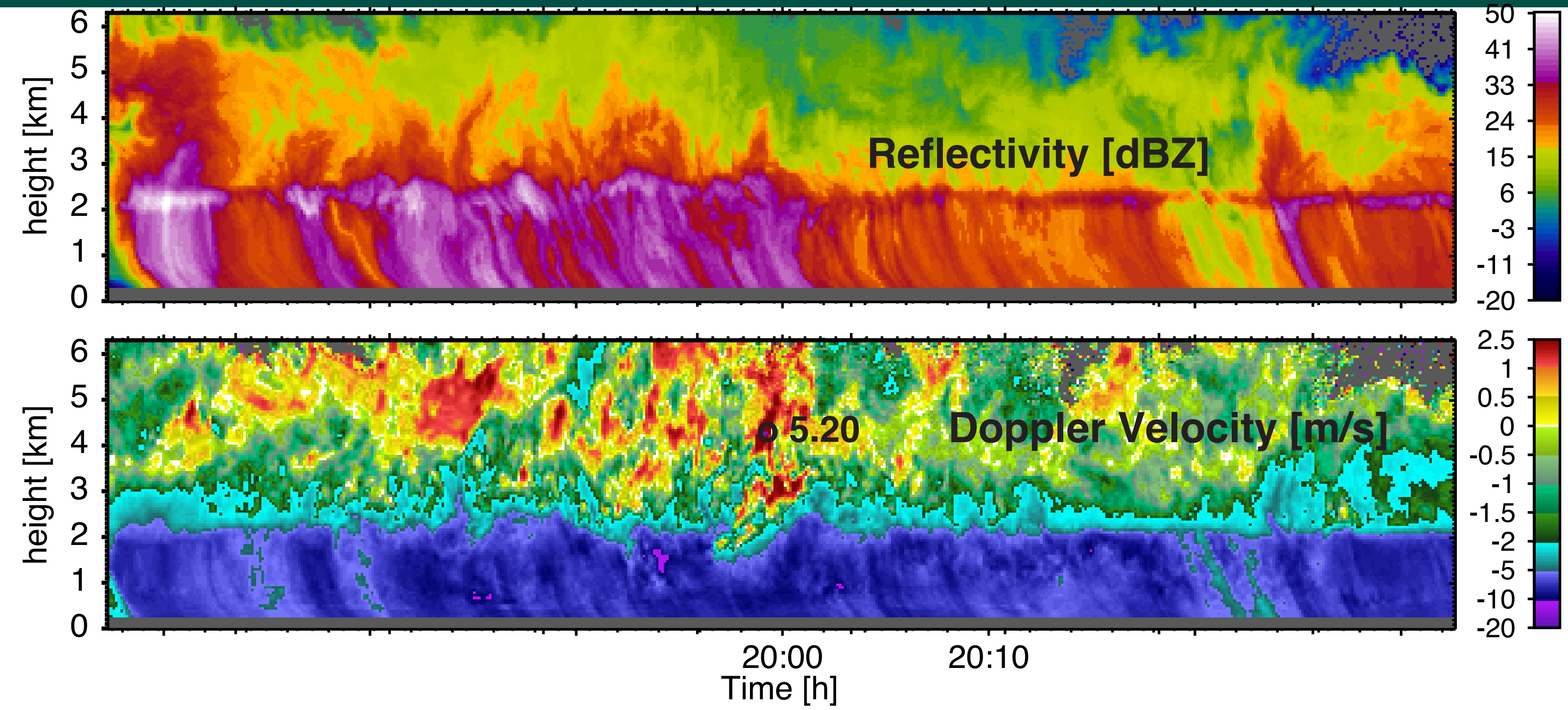
Climatology from 196 convective days of observations (reflectivity exceeded 43 dBZ at some time in the day).

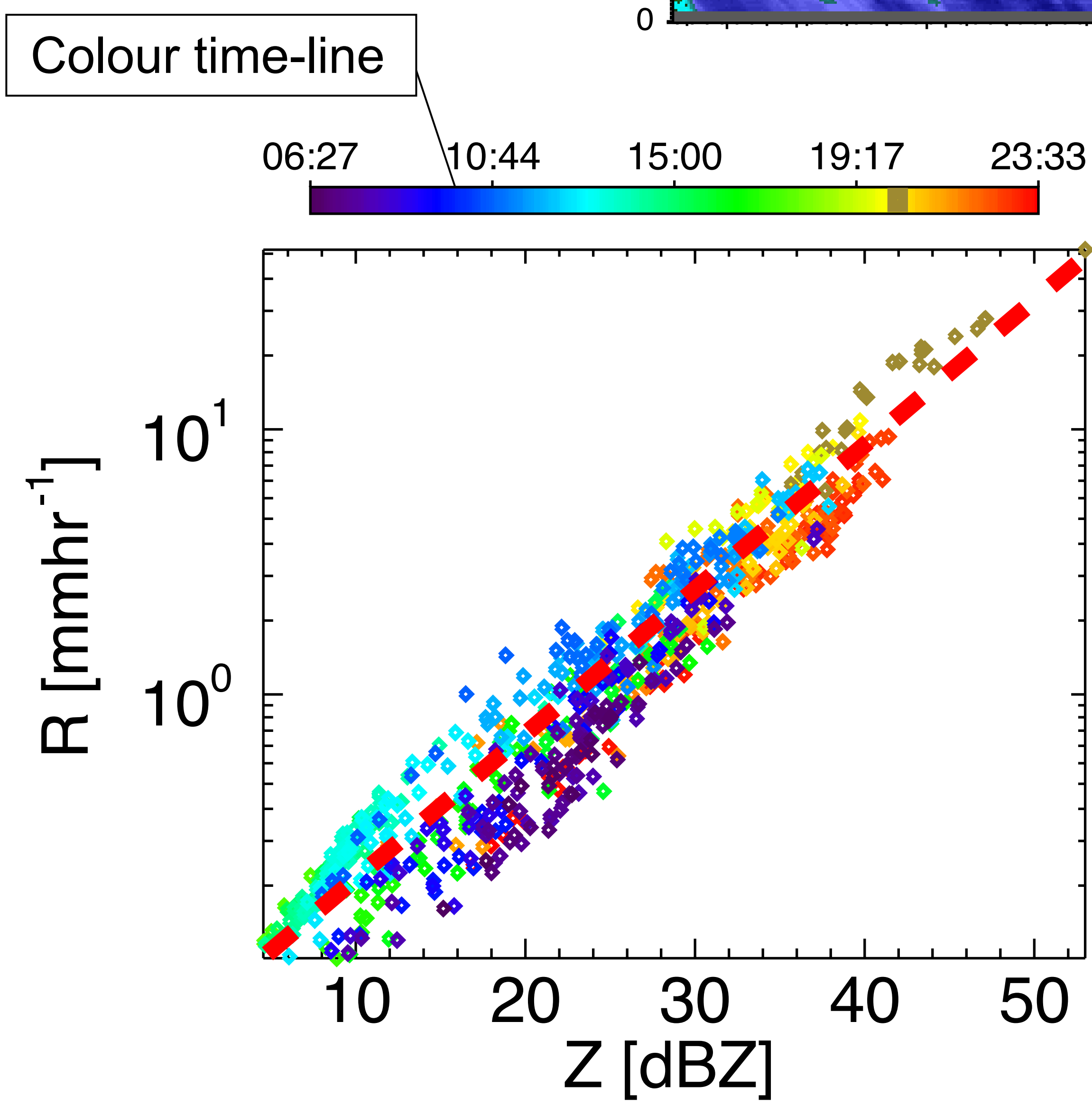
On the variability of DSDs

and its effects on relationship between parameters.

On the variability of DSDs.

A “typical” day of rain: 1June 2004

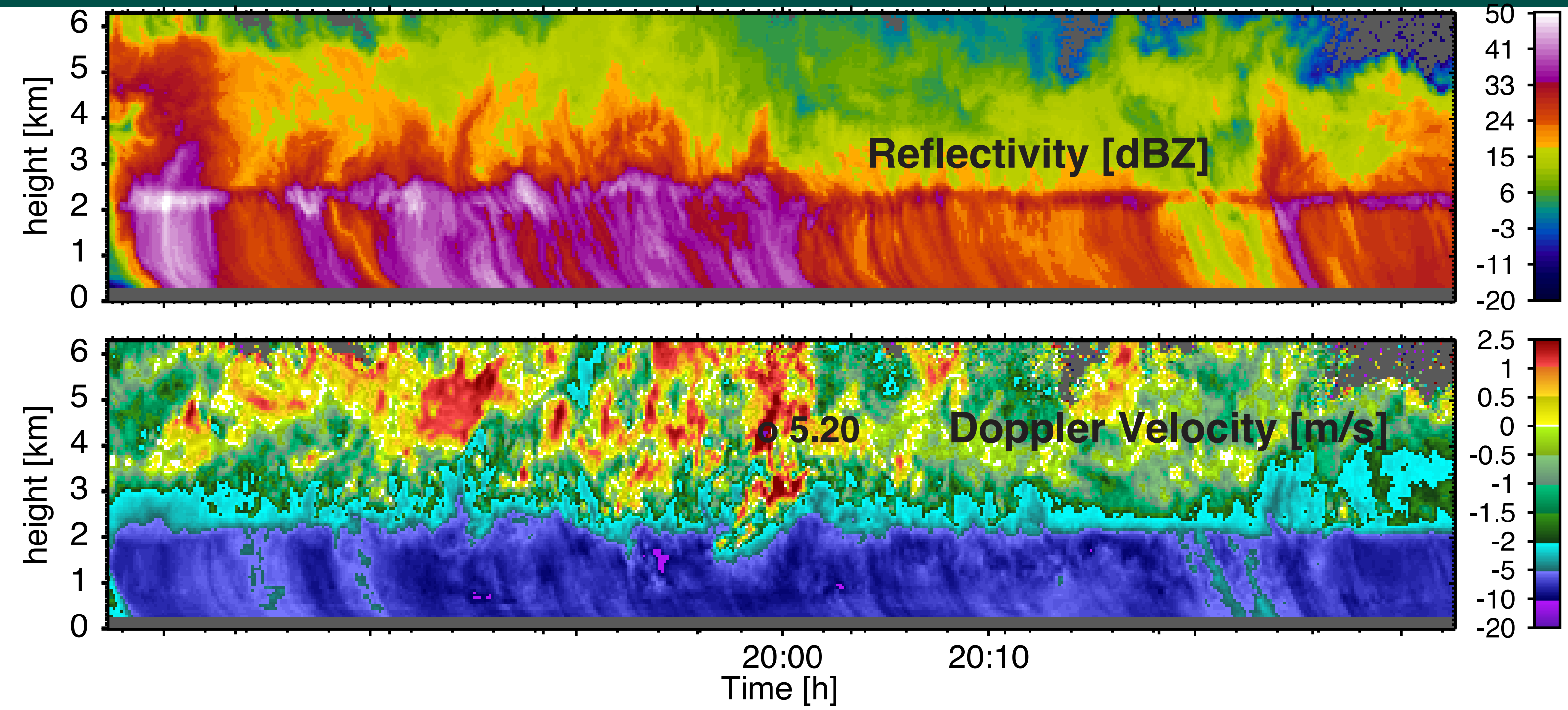


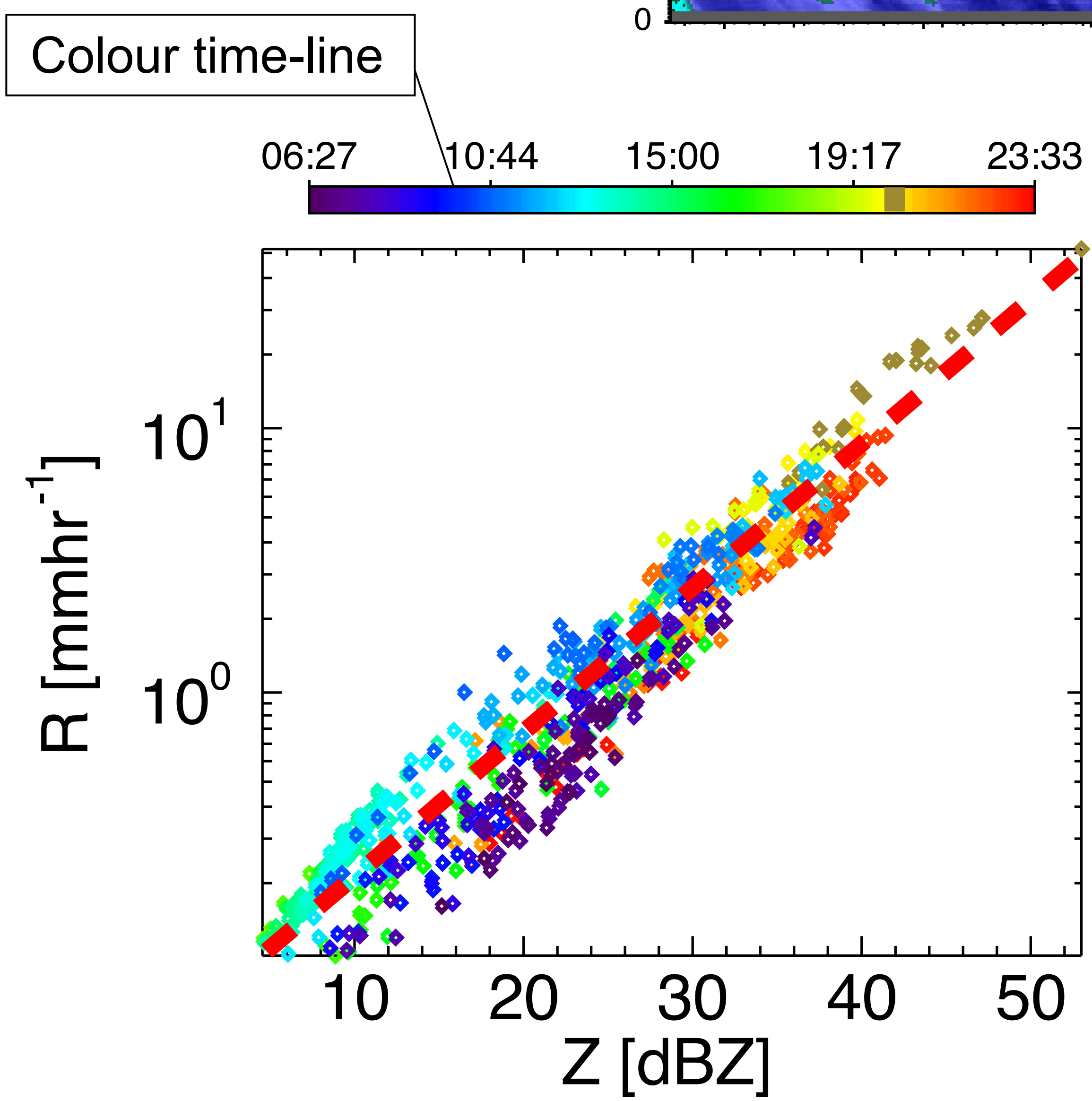


The colour time-line immediately reveals a well organized structure in the stochastic component: some periods (colours) are on one side of the regression other periods are on the other side.

The uncertainty is **not** distributed randomly in space-time. These are structured fluctuations and averaging will not eliminate this type of fluctuations.

On the variability of DSDs.
A “typical” day of rain: 1June 2004





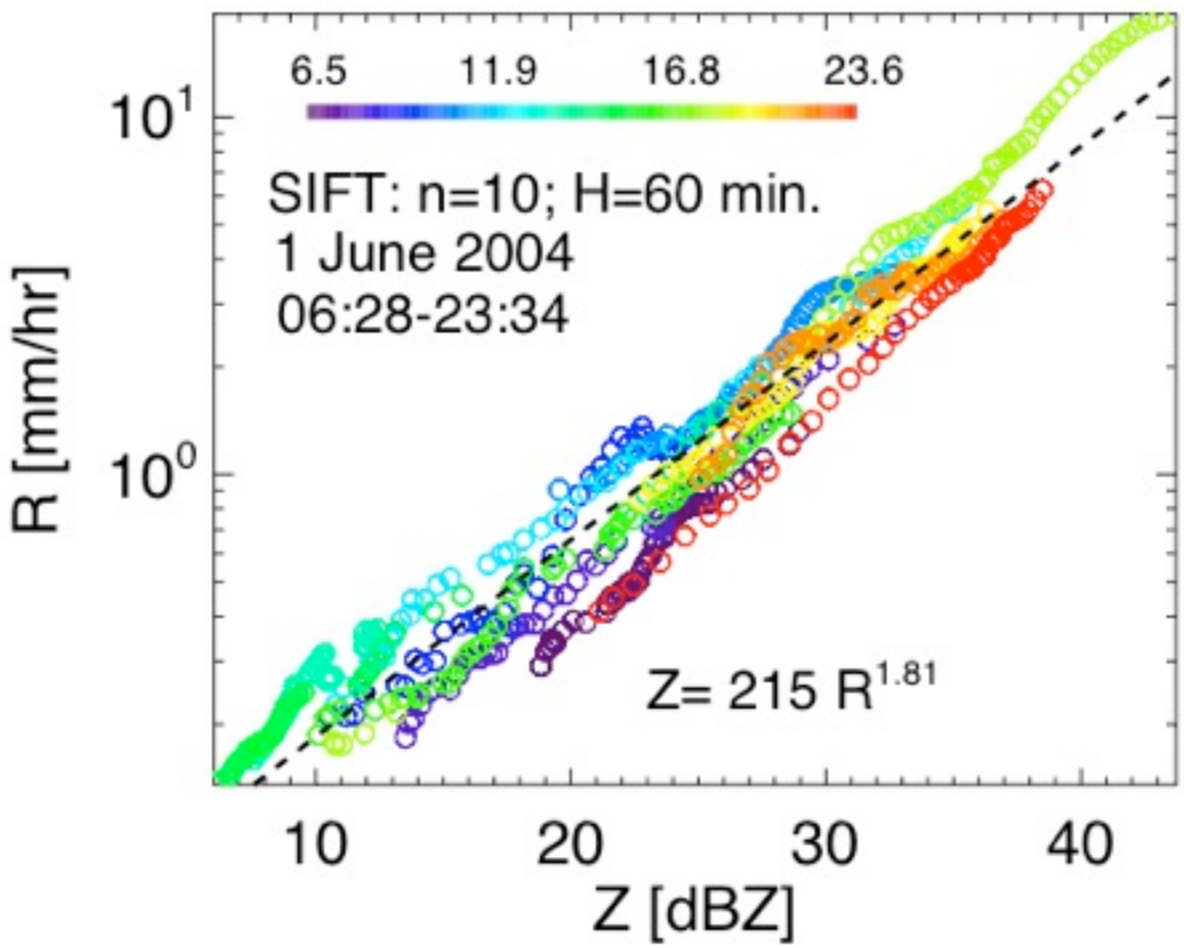
The red regression line represents the “deterministic” (average) relationship and the scatter around this line is the stochastic component (uncertainty) of the R-Z relationship.

The colour time-line immediately reveals a well organized structure in the stochastic component: some periods (colours) are on one side of the regression other periods are on the other side.

The uncertainty is **not** distributed randomly in space-time. These are structured fluctuations and averaging will not eliminate this type of fluctuations.

On the variability of DSDs.
A “typical” day of rain: 1June 2004

Another way of filtering: SIFT. It eliminates the variability within the time window but maintains the R-Z relationships valid within this window.



In the time interval that the storm passed over the disdrometer many Z-R relationships occurred.

As far as we know different regions in every storm go through a good sample of all the possible DSDs and their corresponding Z-R relationships.

Finally apply the most appropriate R-Z and here are samples from a large data base:

These DSDs are also averaged in a similar manner: within a one hour interval groups of 10 DSDs of consecutive reflectivity are averaged

There is a great degree of variability in the DSDs. We have a reasonable understanding of the microphysical processes that cause these various distributions.

For example: Distributions in very heavy rain:

Equilibrium DSDs

The evolution of DSDs can be expressed by a general simple form as:

$$\frac{\partial N(d)}{\partial t} = \int_{d/2}^{\infty} \int_{d-x}^{x} K(d;x,y)n(x)n(y)\,dx\,dy$$

where $K(d;x,y)$ is a function that represents the complex drop interactions, coalescence and break-up, leading to changes in the distribution.

When the number of drops of a given size is created by one of the processes is exactly compensated by the destruction caused by the other process we have equilibrium:

$$0 = \int_{d/2}^{\infty} \int_{d-x}^{x} K(d;x,y)n(x)n(y)\,dx\,dy$$

If $N(d)$ is a solution of this equation $k\,N(d)$ is also a solution; the distribution are proportional (parallel) to each other when intensity changes.

Equilibrium DSDs

Therefore we have:

$$Z_1 = \int D^6 N(d) dD \qquad R_1 = c_0 \int V(D) D^3 N(d) dD = c \int D^{3.6} N(d) dD$$

$$Z = \int D^6 k N(d) dD = k Z_1 \qquad R = c \int D^{3.6} k N(d) dD = k R_1$$

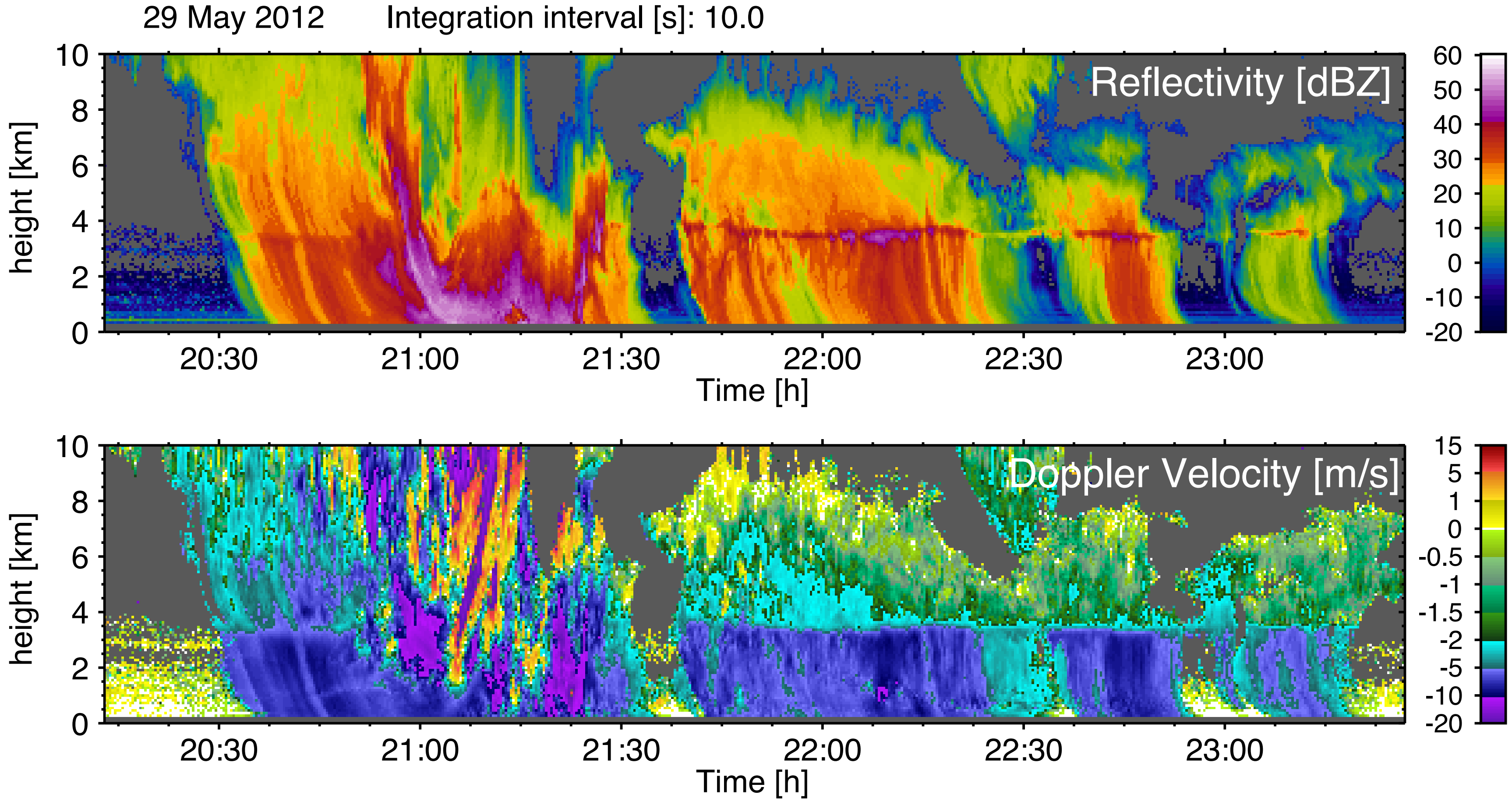
$$\frac{R}{R_1} = \frac{Z}{Z_1} \quad \Rightarrow \quad R = \frac{R_1}{Z_1} Z$$

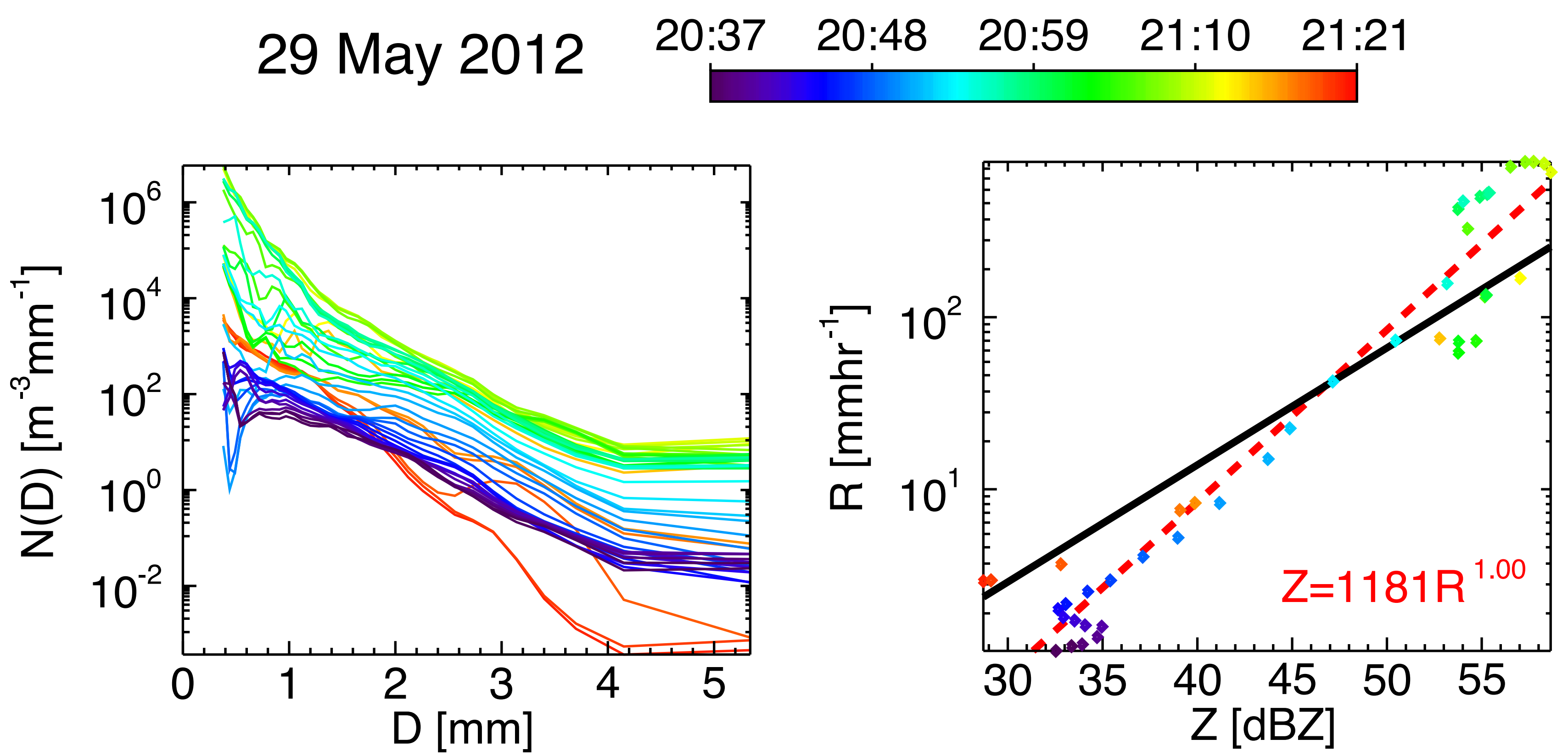
This shows that for distributions that are parallel to each other, when intensity changes Z and R change proportionally!

There are other mechanisms that may lead to equilibrium (not as well studied as the previous), such as snow aggregation and snow growth by deposition or droplet growth by cloud collection and cloud transfer to rain by autoconversion.

Does it really exists?

Yes, even in Montreal



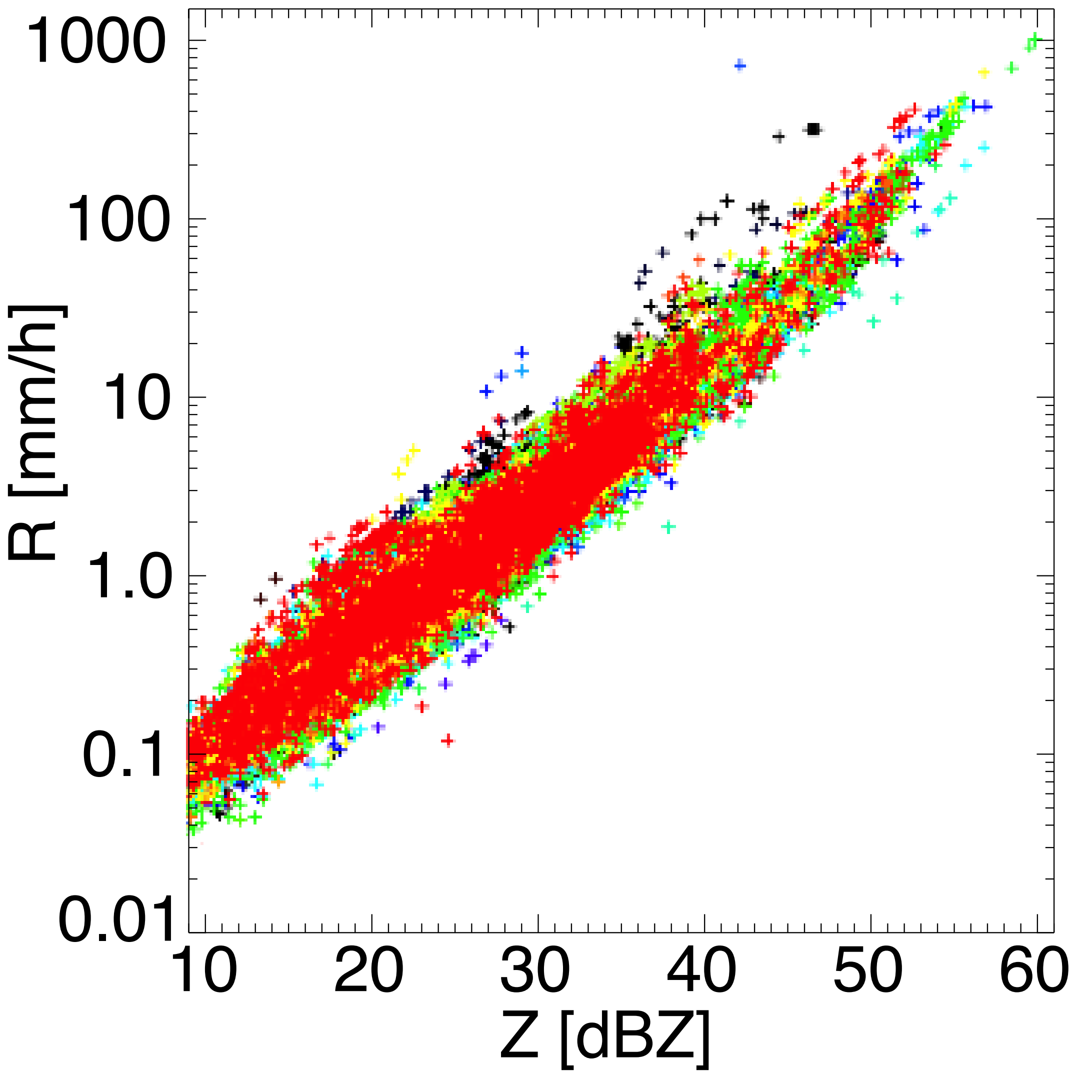


So, why do we keep using a power-law Z-R relationship that is not even compatible with the physics of DSD formation that we understand and know?

Let me suggest an alternative approach based on the stochasticity assumption:
because of the stochastic variability in time and space of the microphysical processes that shape the DSDs the fluctuations in DSDs are stochastic.

The ergodicity hypothesis for stochastic processes states that a time sequence of observations can be considered as a good representation of the process.

Climatology



Climatology from 196 convective days of observations (reflectivity exceeded 43 dBZ at some time in the day).

Thus, let us consider the formation of the DSDs as a stochastic process and each observation of DSD as one realization of this process. By the ergodicity hypothesis our observations in time describe the stochastic process.

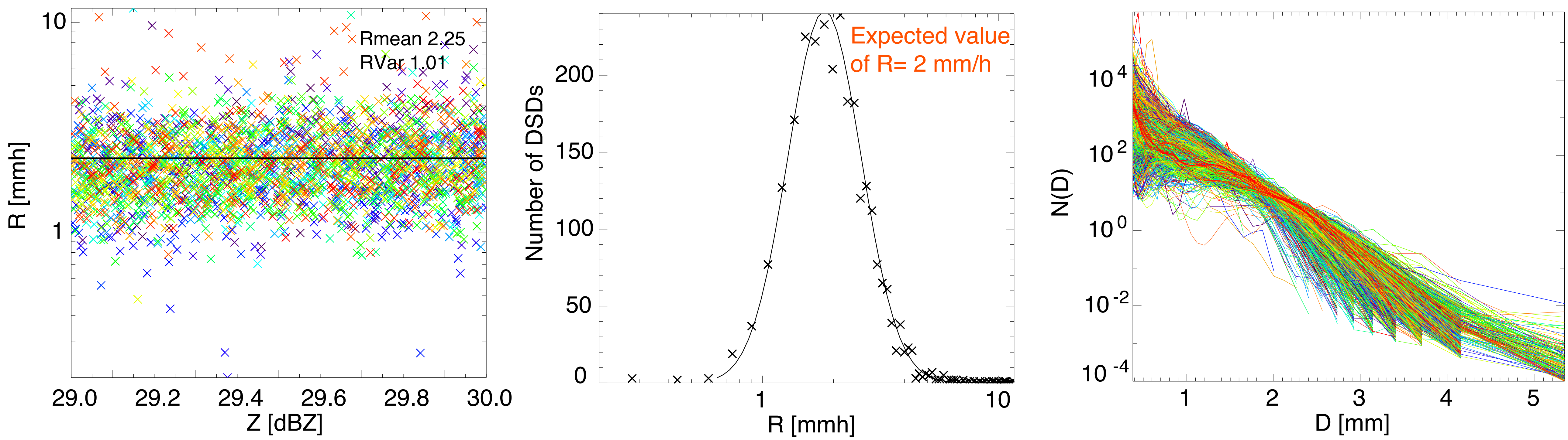
Distributions as stochastic process:

Here we formulate the retrieval of R from Z as

$$\boxed{R = \int r \; p(r \mid [Z \pm \delta]) \, dr}$$

Let us take $\delta = 0.5 \text{ dB}$

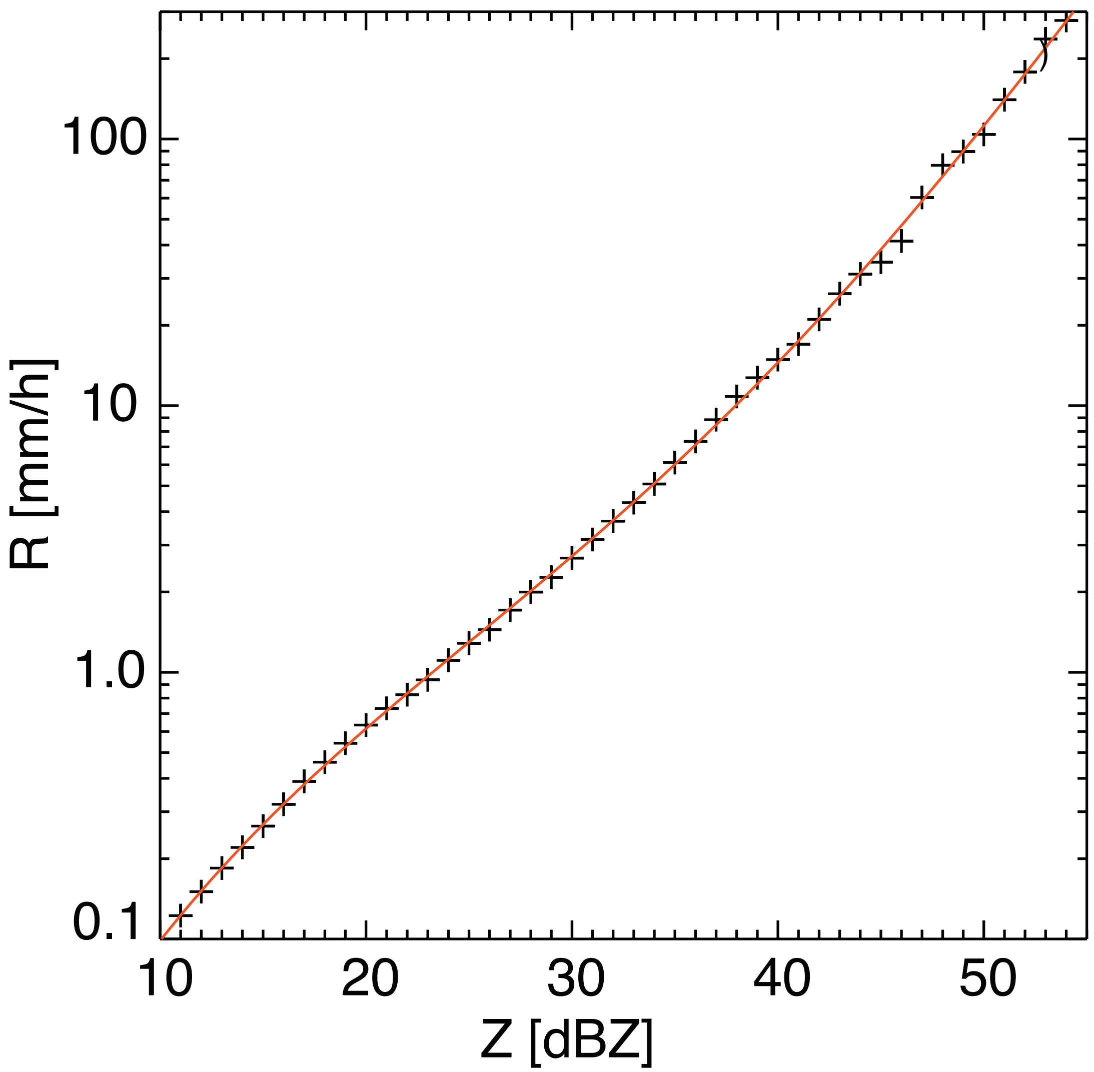
and for each dBZ interval determine $p(r \mid Z)$



$$R = \int r \; p(r \mid [29 - 30 \, dBZ]) \, dr = 2$$

Expected Value of R

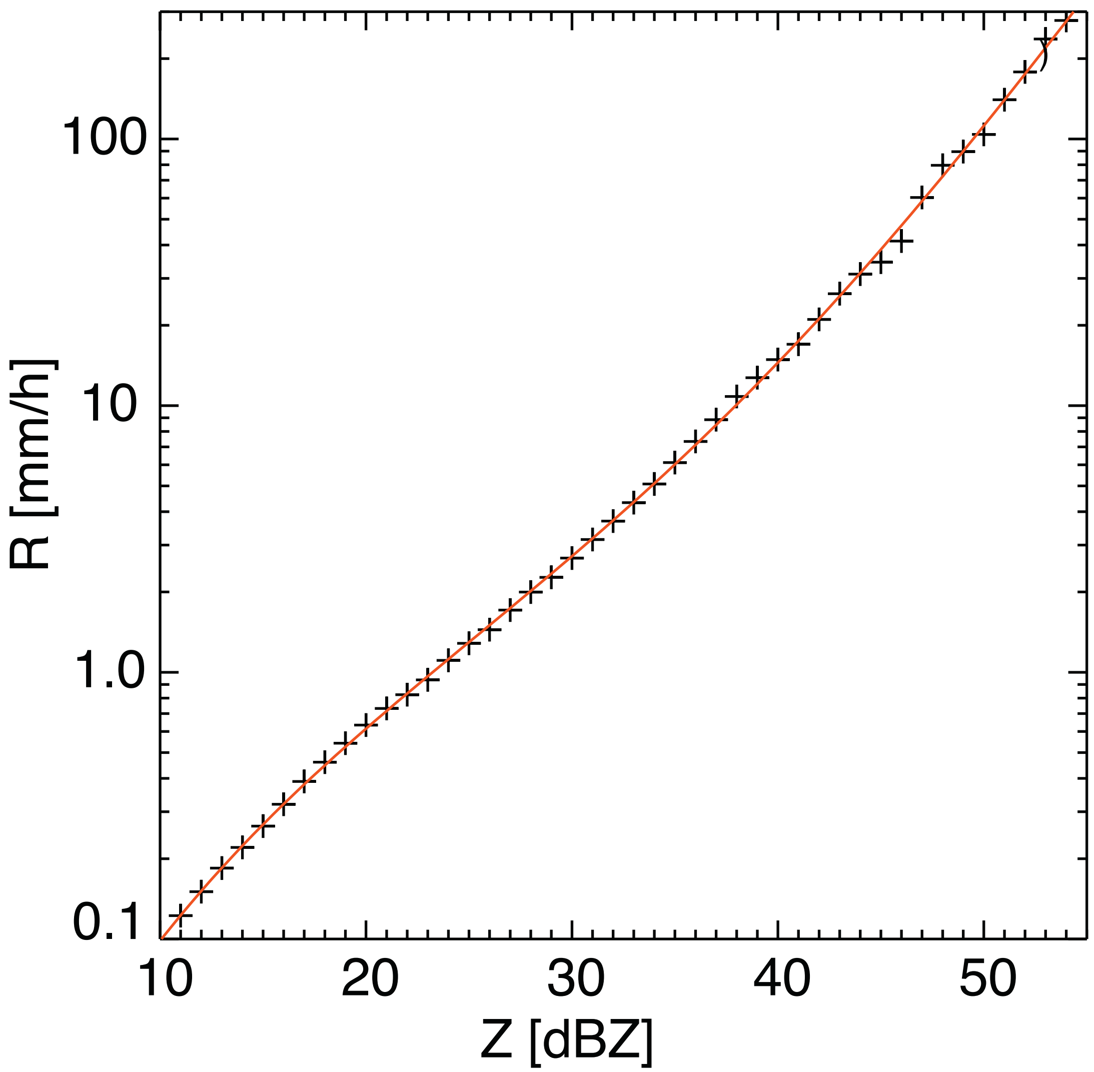
From our sample of 196 days of disdrometric records we have this expected value of logR versus logZ



Expected Value of R

From our sample of 196 days of disdrometric records we have this expected value of logR versus logZ

$$\log R = -2.3+0.17Z-5.1\times10^{-3}Z^2 +9.8\times10^{-5}Z^3-6\times10^{-7}Z^4$$



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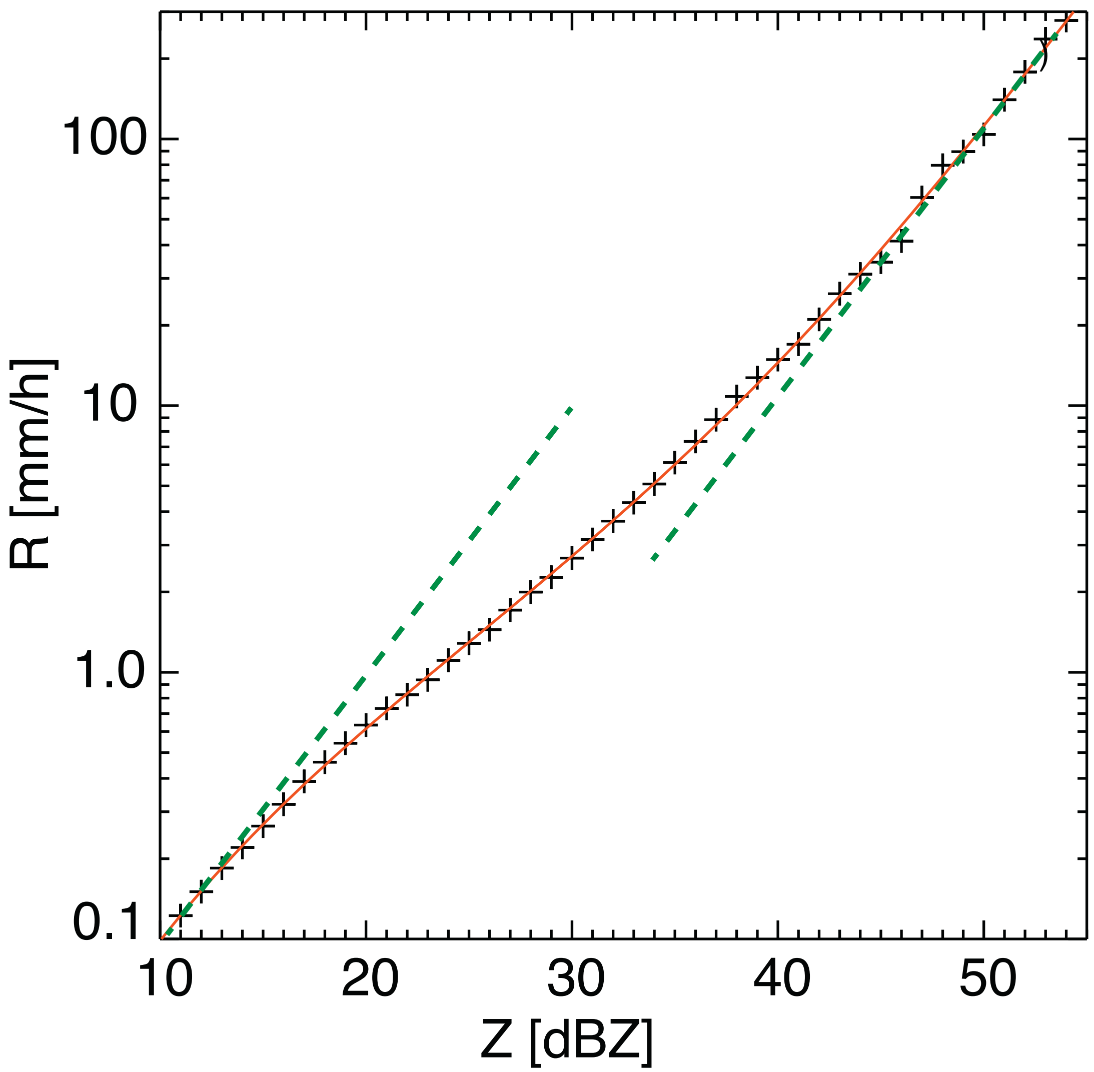

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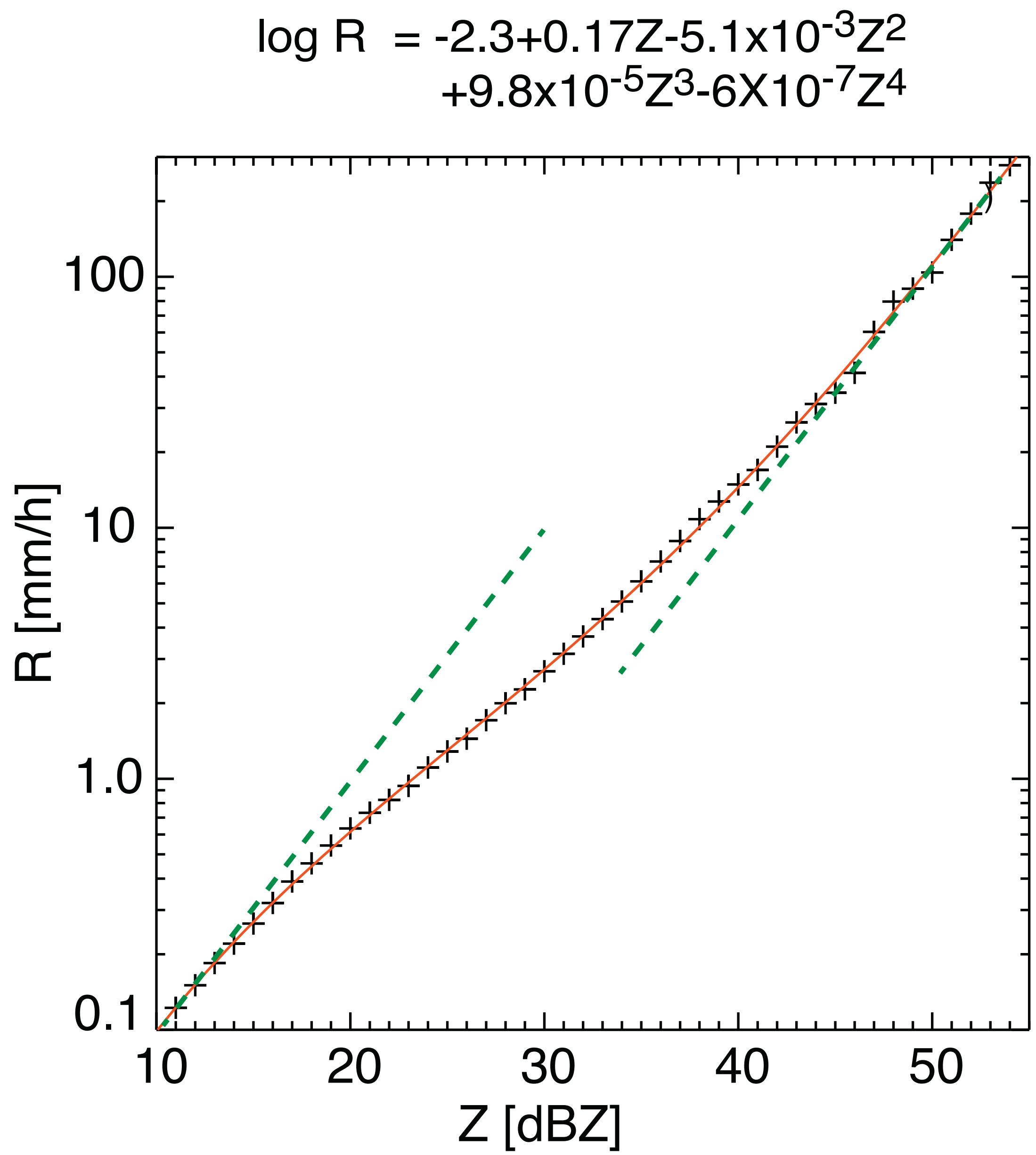

Some small progress here!

We get a more complex relationship that in fact has some physical sense:
it is consistent with the tendency to equilibrium DSDs at Z>40dBZ and the expected behaviour at very low rates where cloud collection is the prevailing mechanism of precipitation growth

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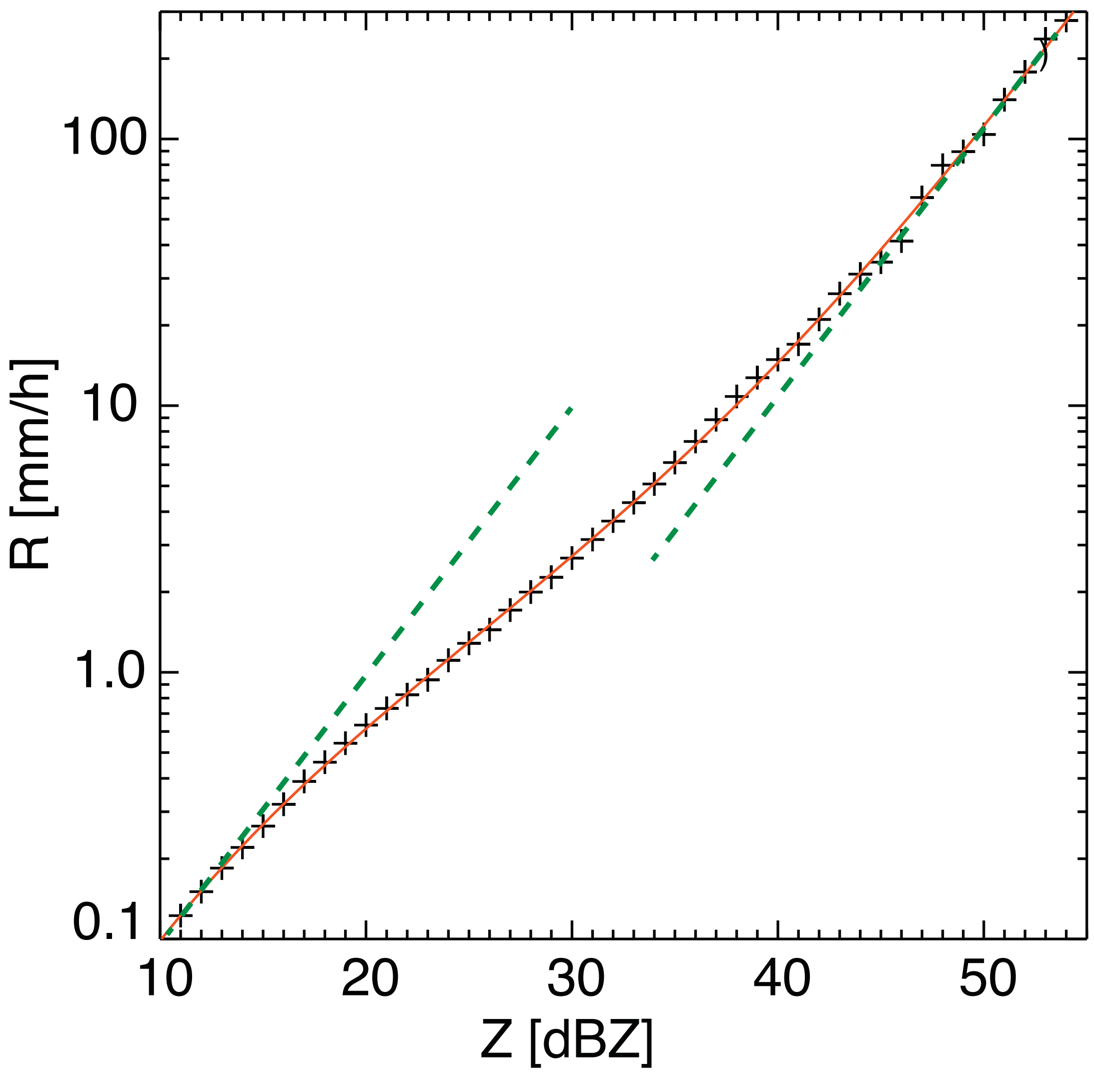

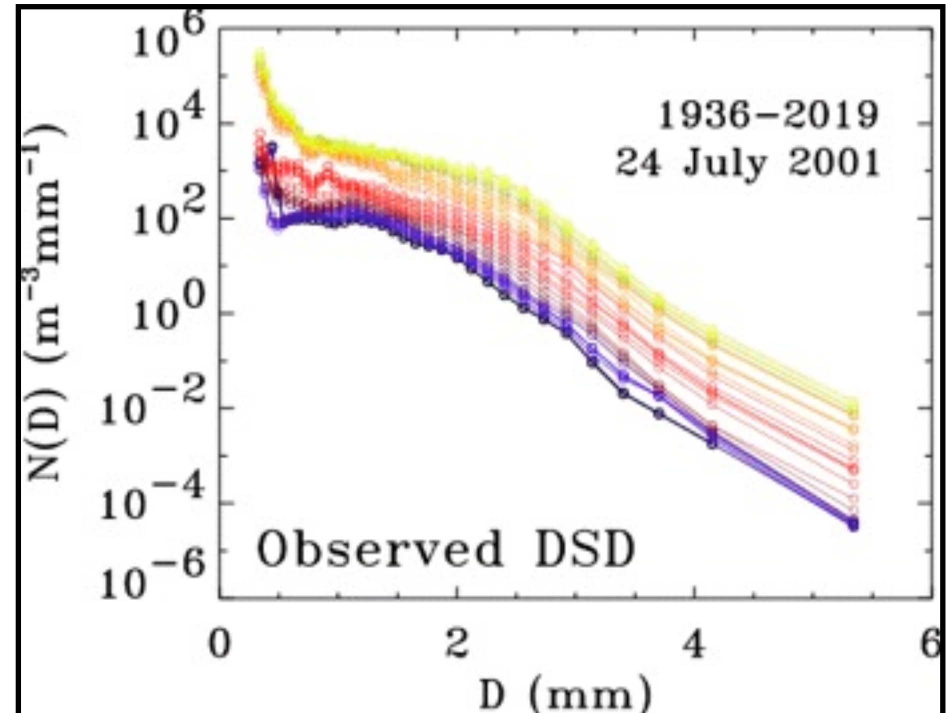

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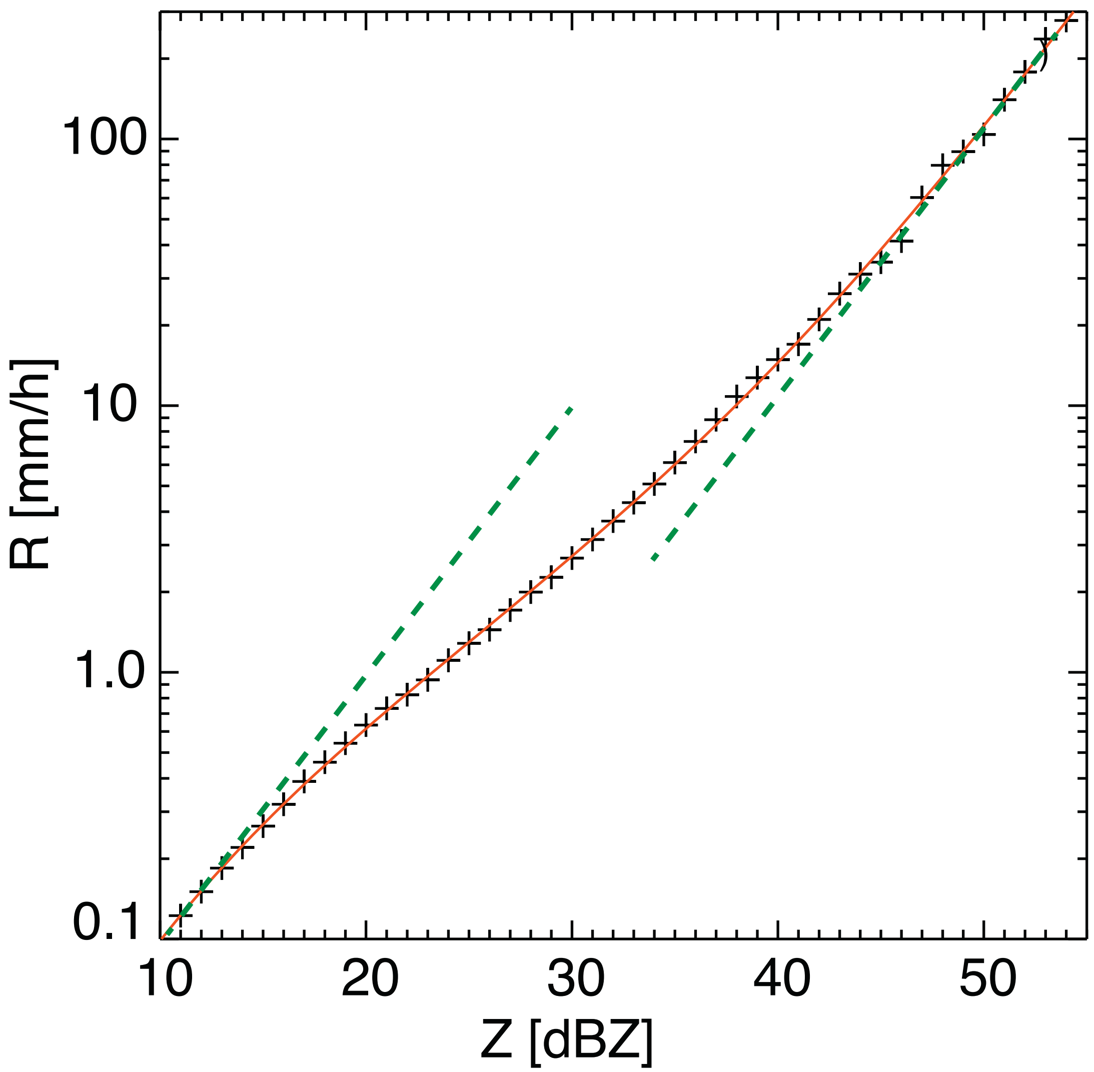

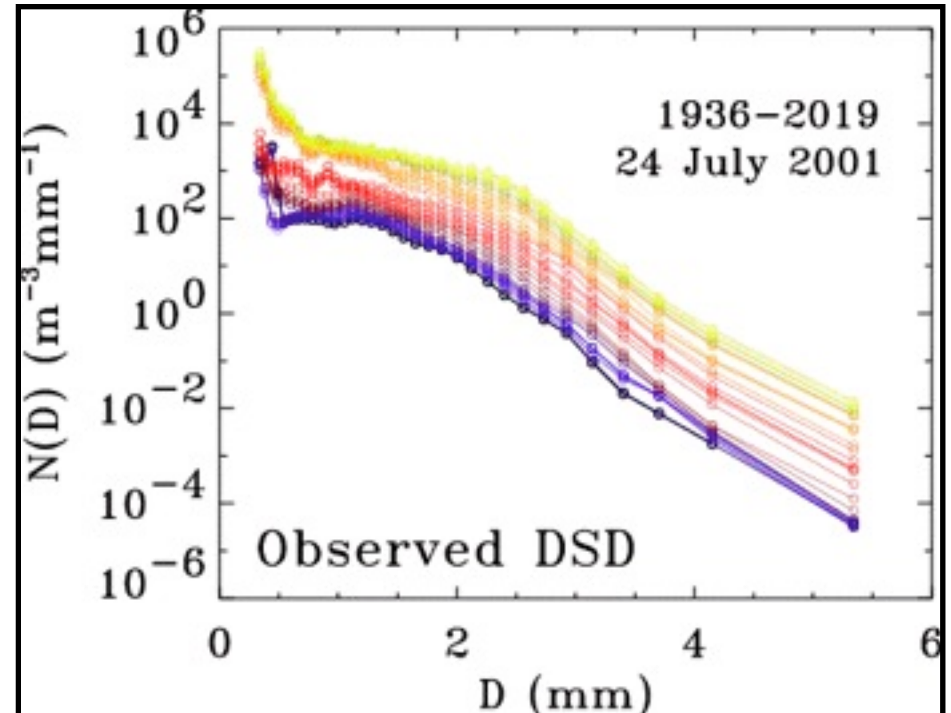

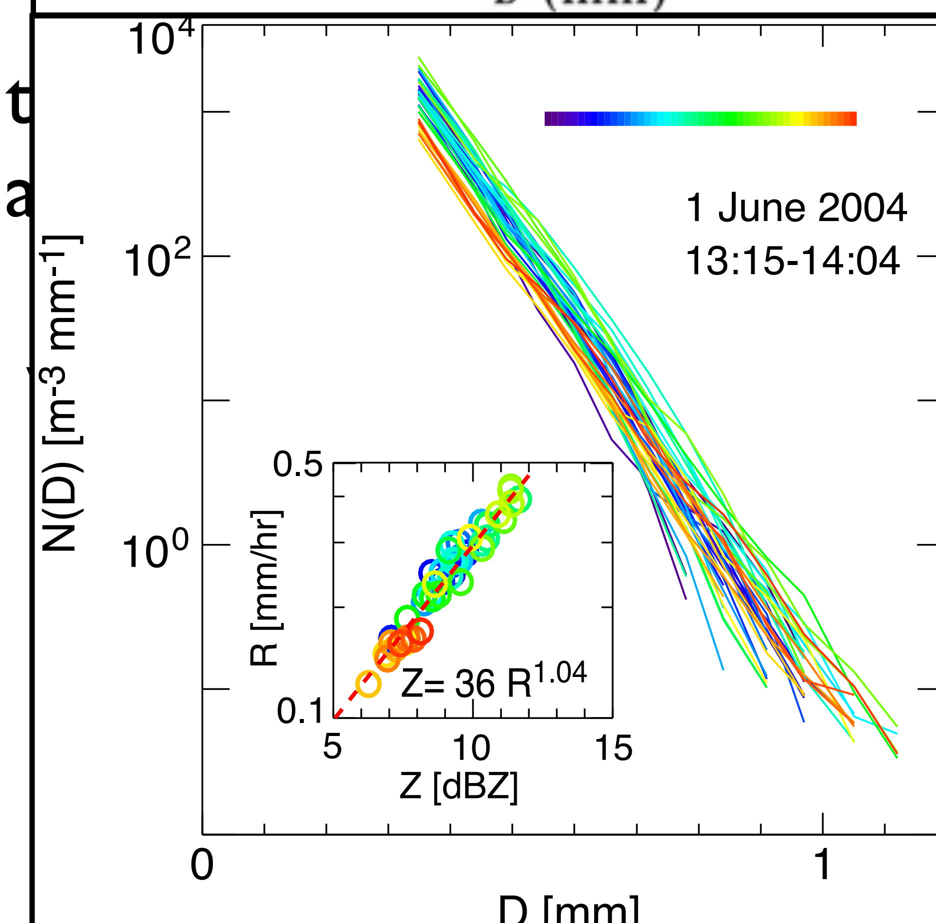

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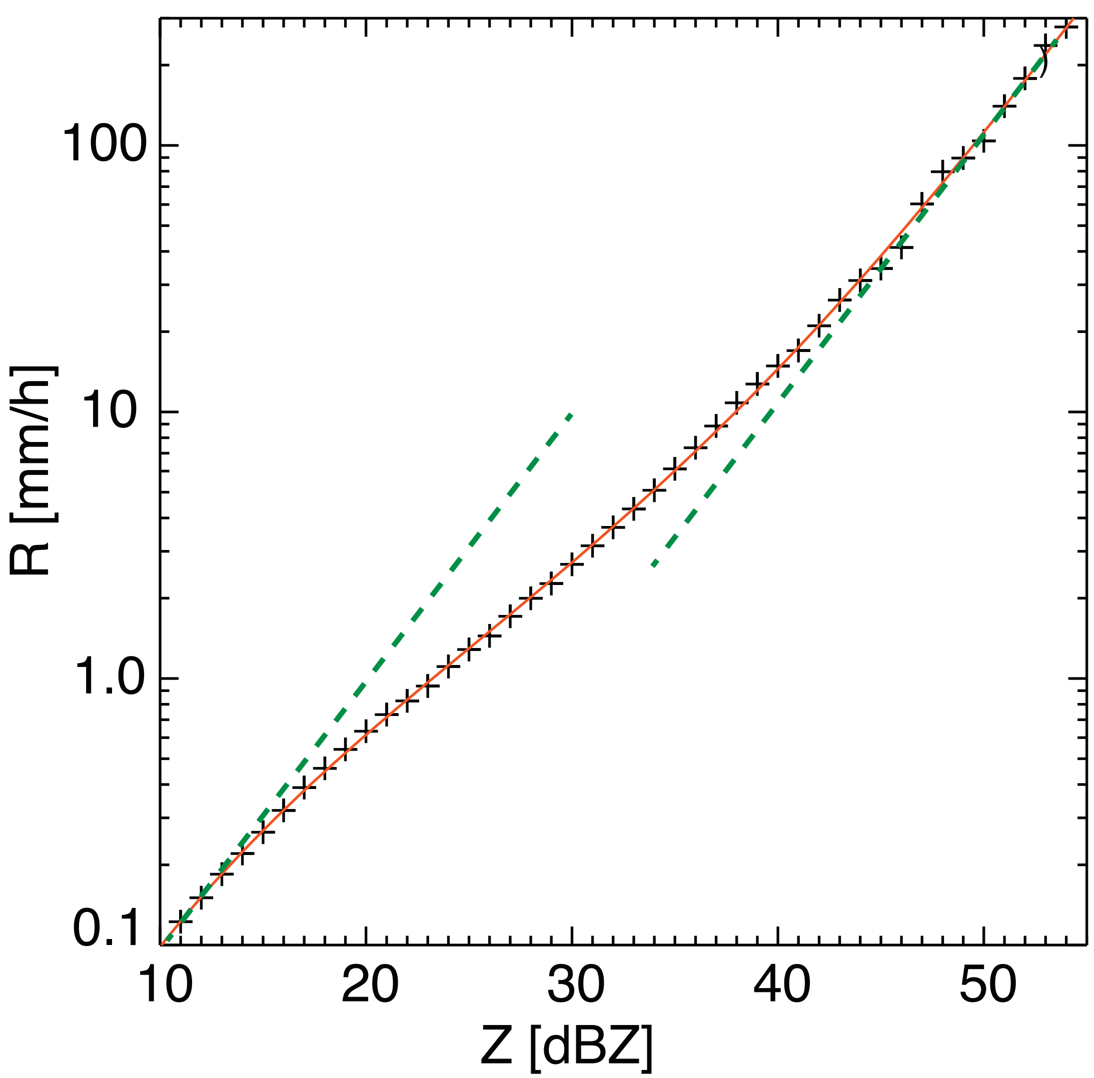
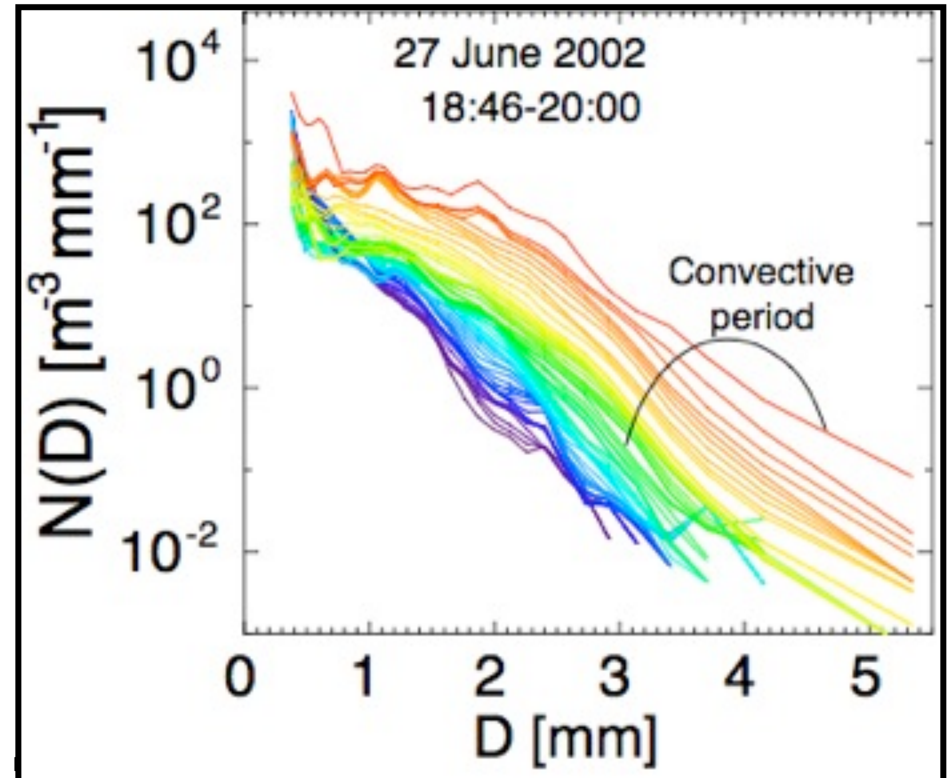
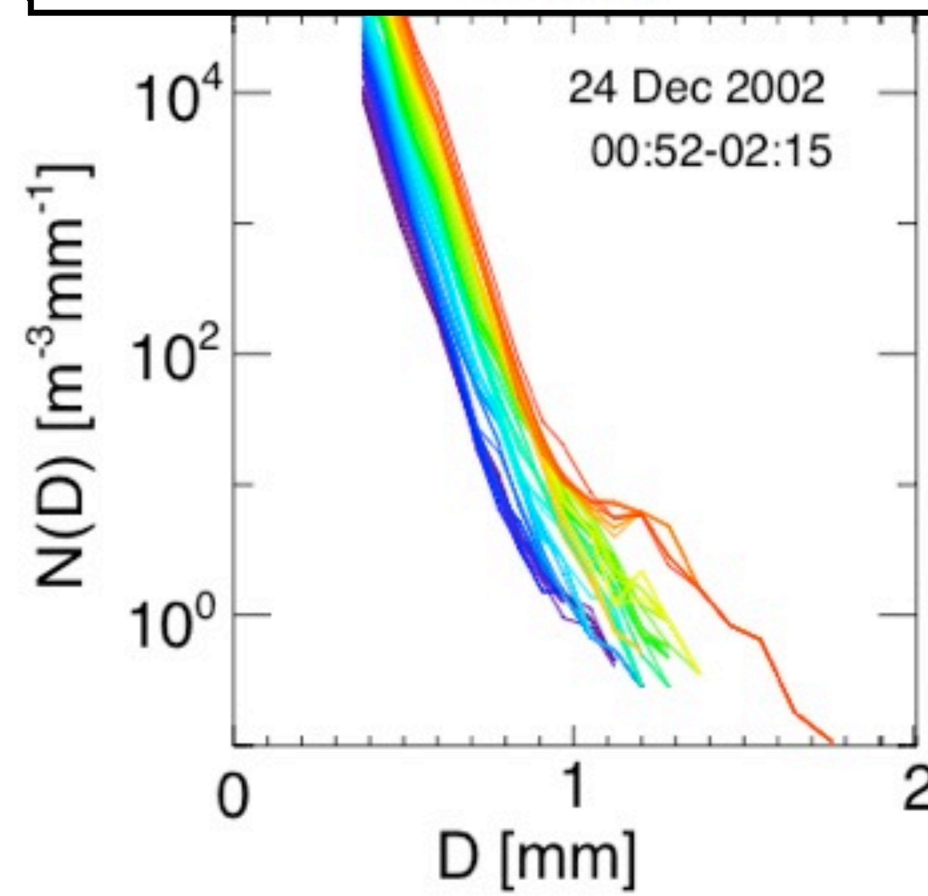
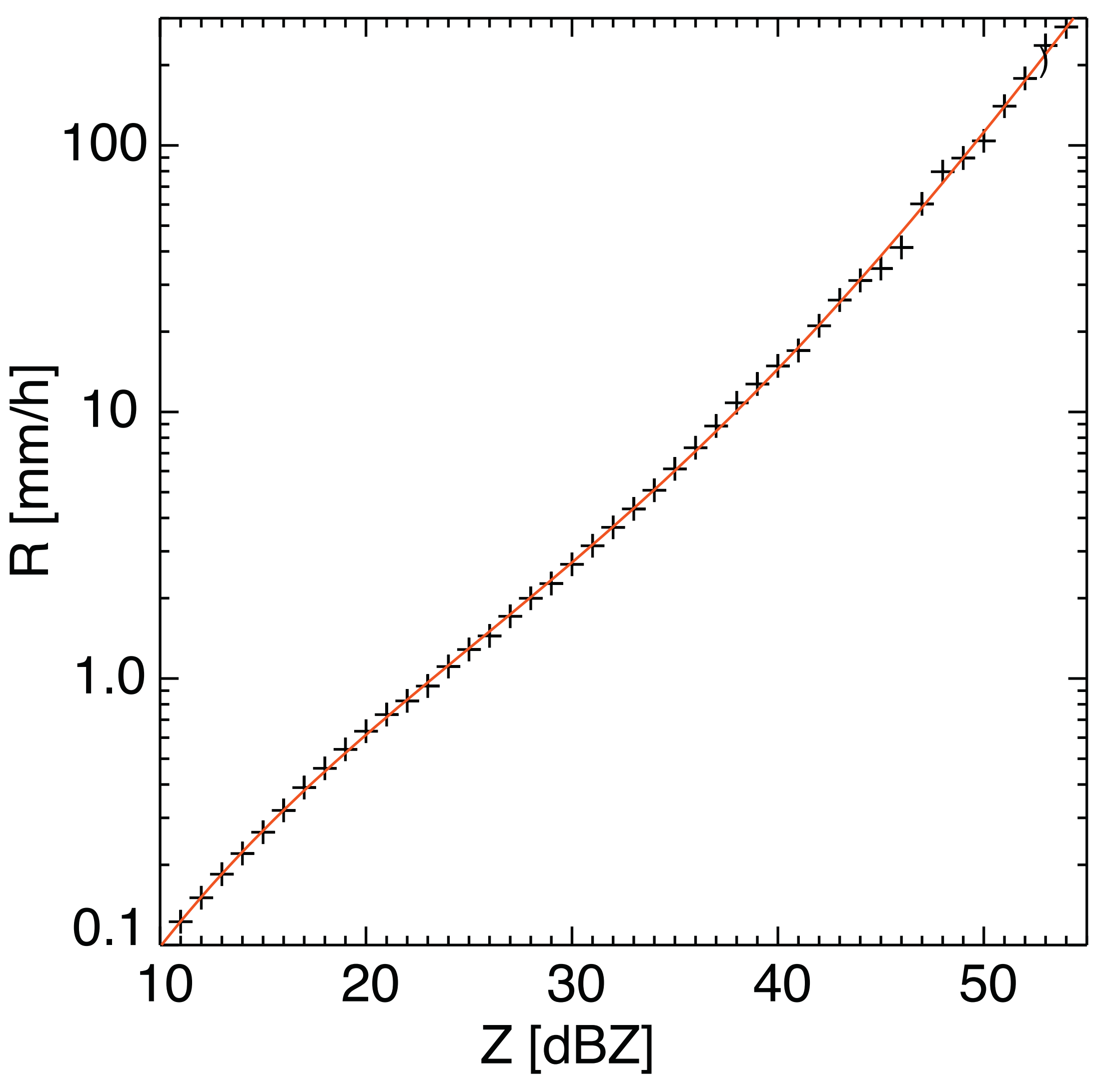


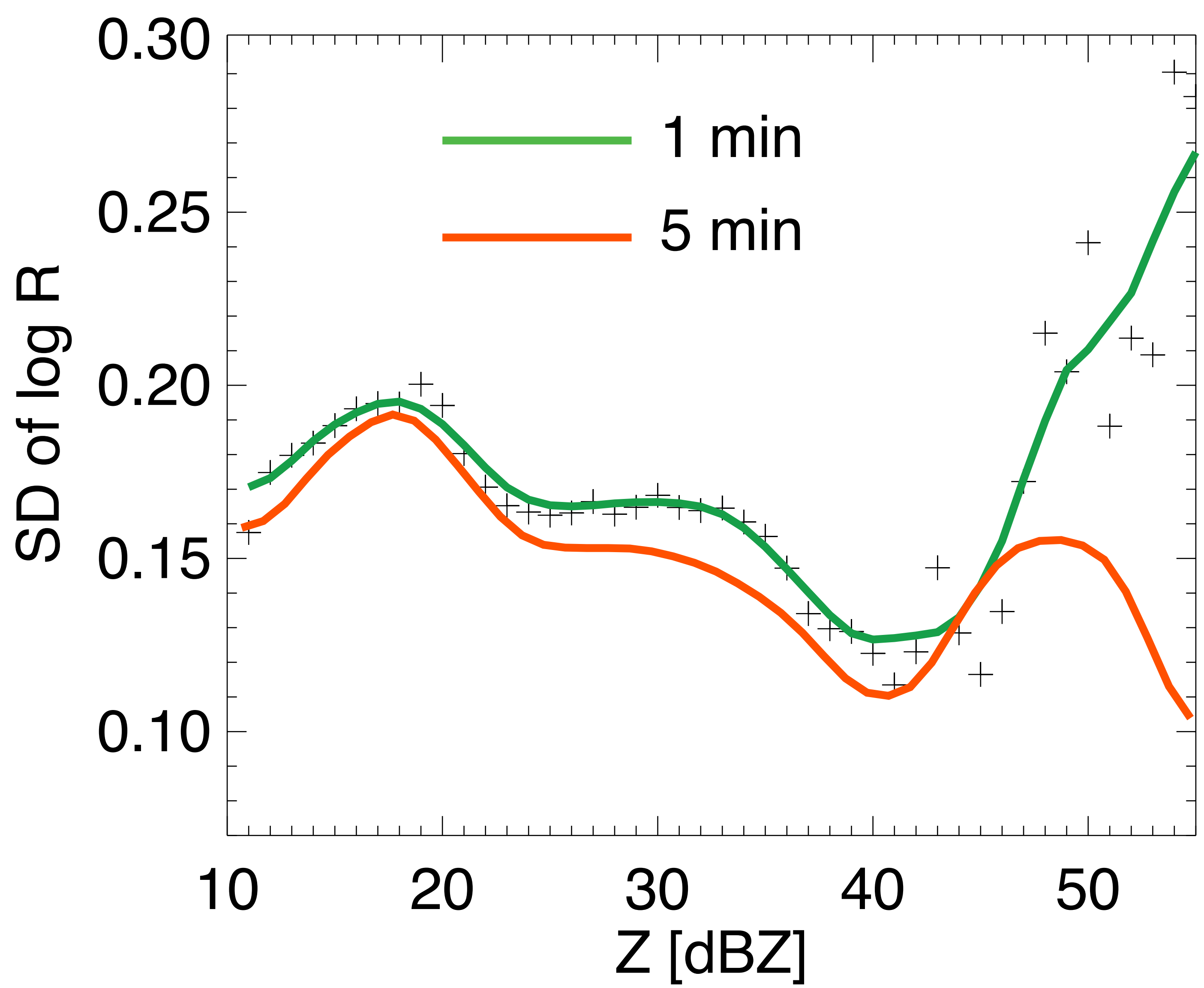




Expected Value of R

From our sample of 196 days of disdrometric records we have this expected value of logR versus logZ and its standard deviation





From our sample of 200 days of disdrometer records we have this expected value of logR versus logZ

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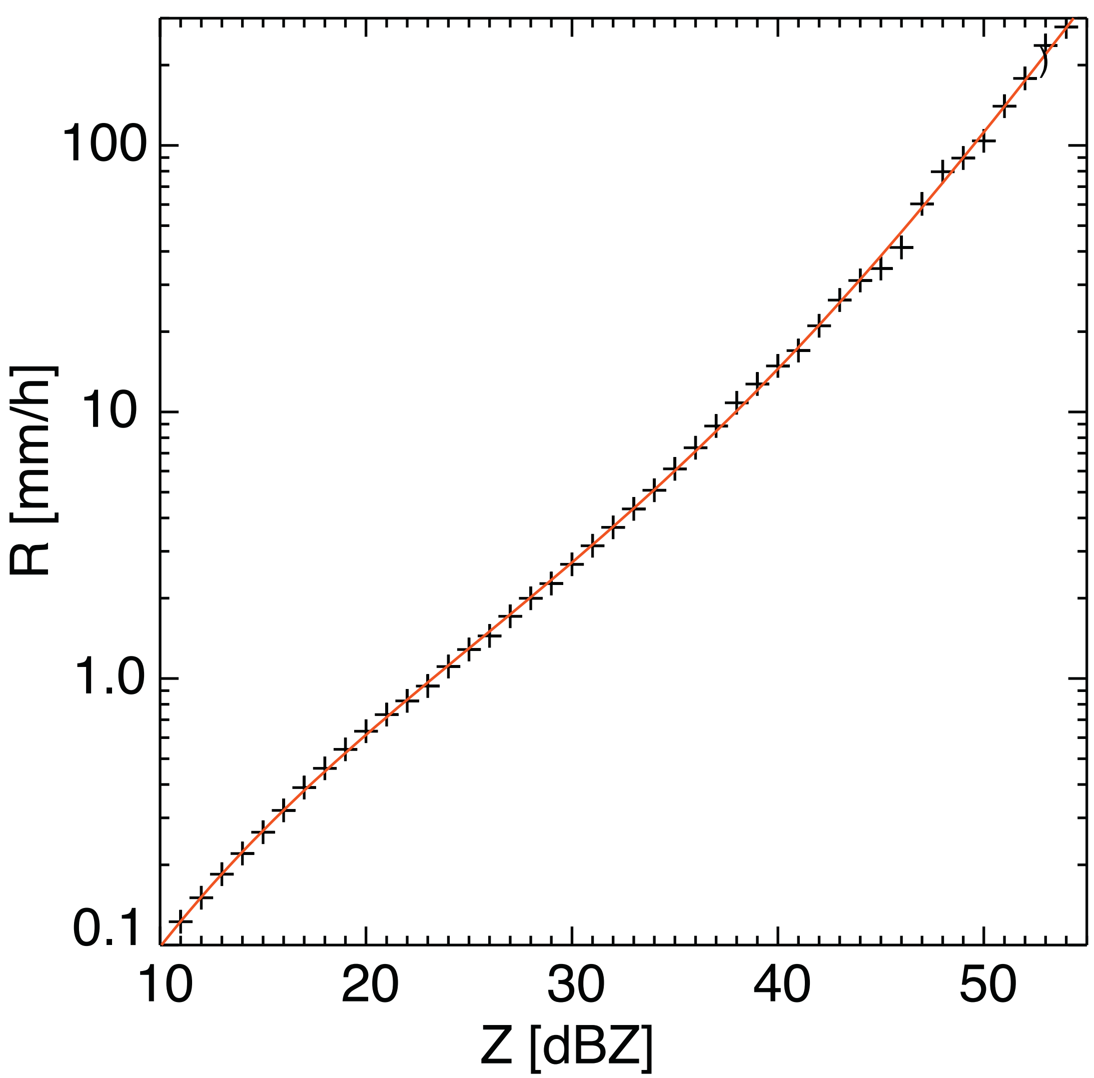

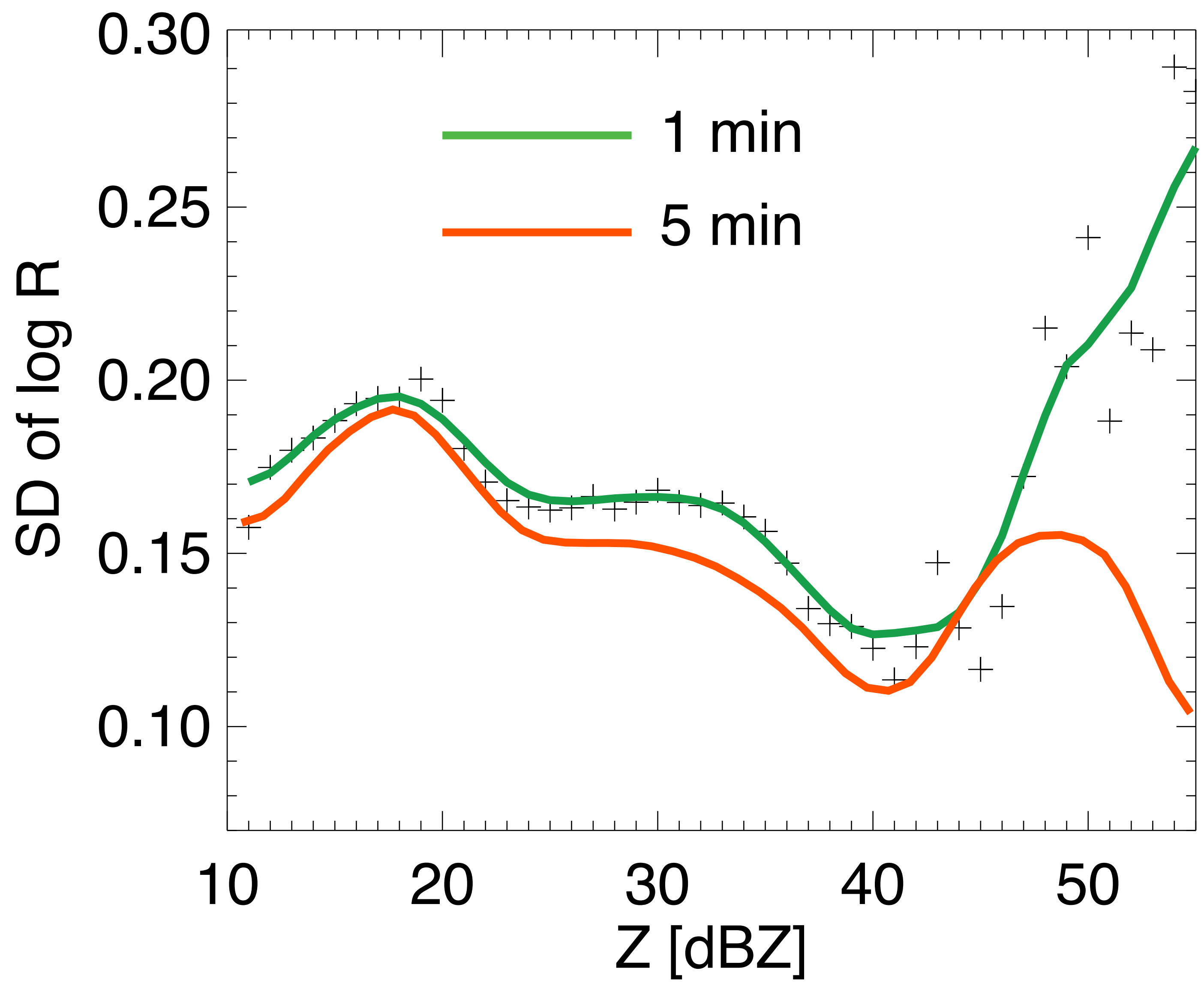

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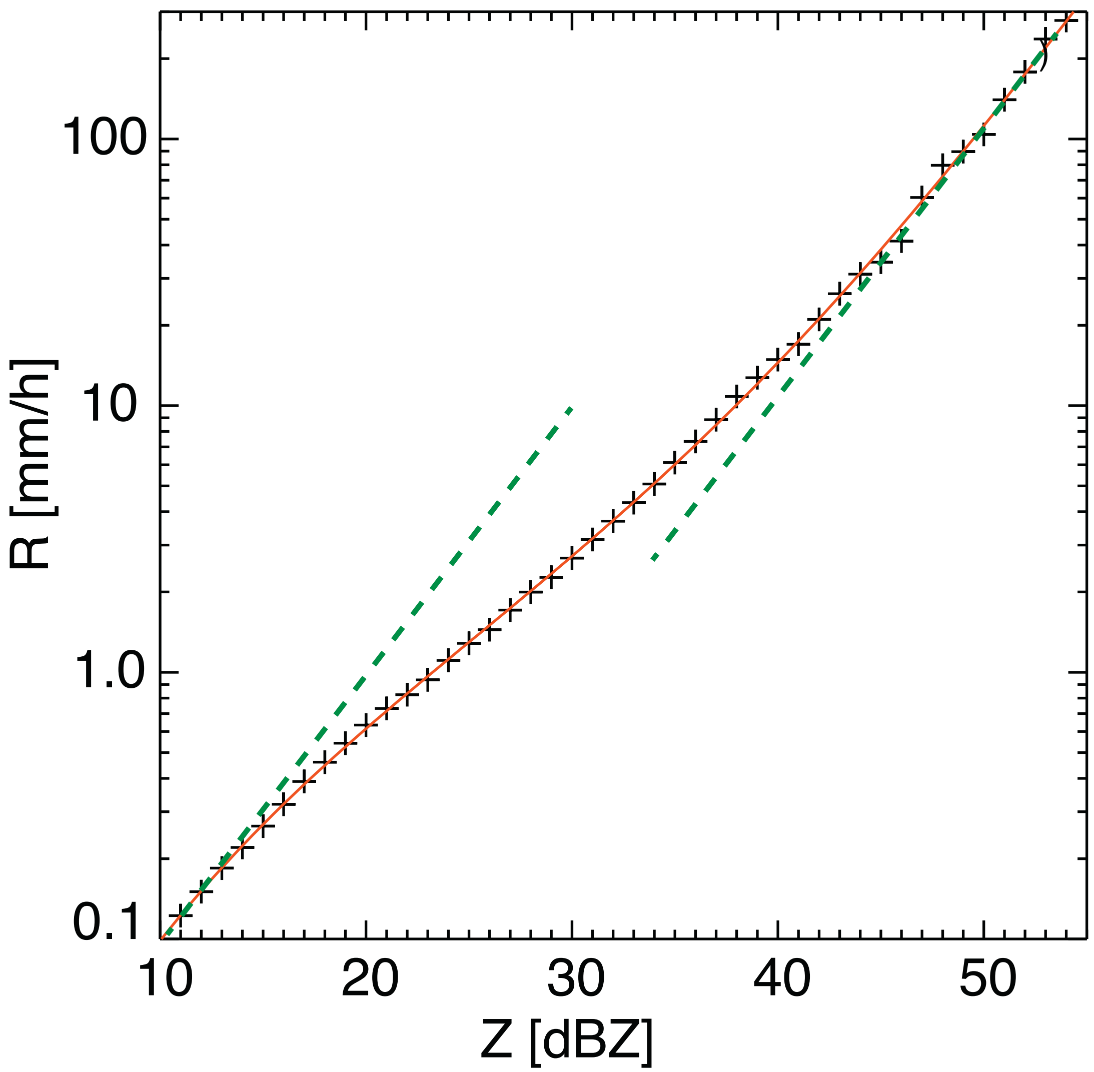

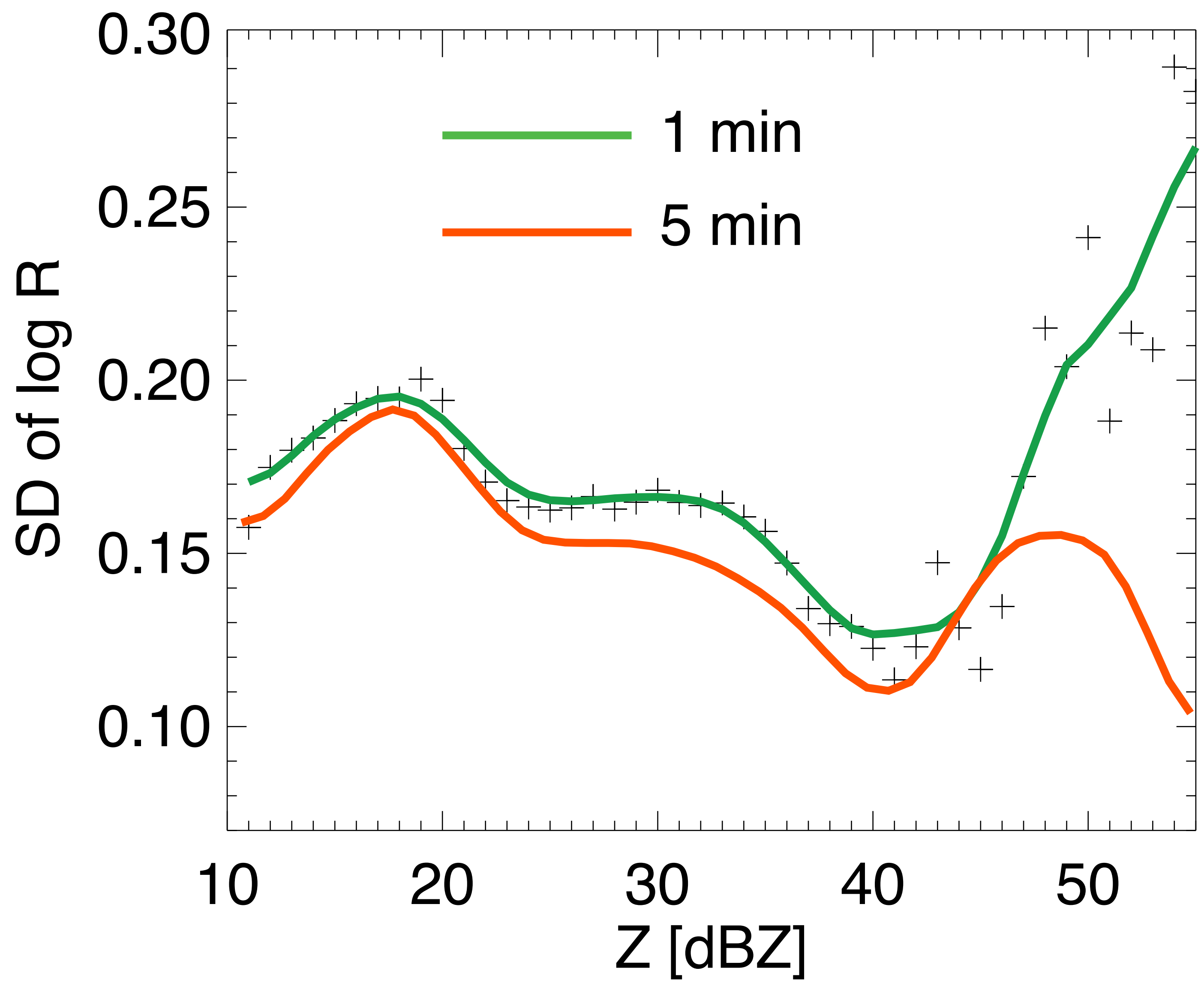

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