ACCOUNTING FOR AND CORRECTING MODEL ERRORS IN ENSEMBLE KALMAN FILTER

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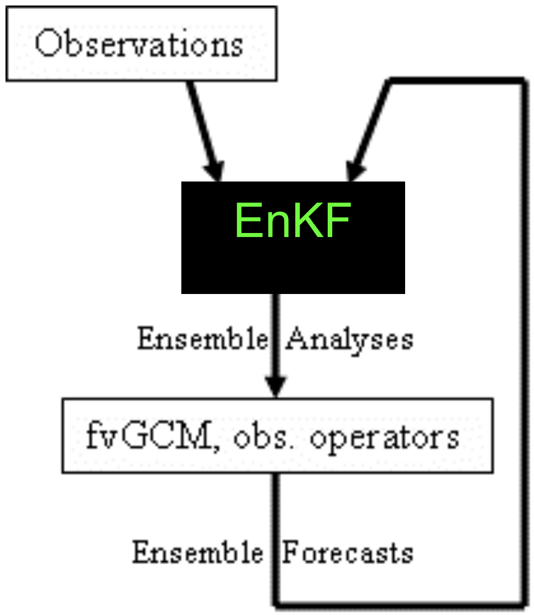
Motivation (1)

- Ensemble based Kalman Filters (EnKF) are emerging as a potential replacement for operational 3- or 4-dimensional variational data assimilation system, because:

1) They know about the 'errors of the day'

$$\mathbf{x}_a = \mathbf{x}_f + \mathbf{K}[\mathbf{y}_0 - h(\mathbf{x}_f)] \qquad \mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}^T [\mathbf{H}\mathbf{P}_i^f \mathbf{H}^T + \mathbf{R}]^{-1}$$

$$\mathbf{P}_i^f \approx \frac{1}{k-1} \sum_{i=1}^{K} (\mathbf{x}_i^f - \overline{\mathbf{x}^f})(\mathbf{x}_i^f - \overline{\mathbf{x}^f})^T$$



2) They are very simple to code and implement (model independent, do not require adjoint of model)

3) Automatically provide an ensemble of states to initialize ensemble forecasts

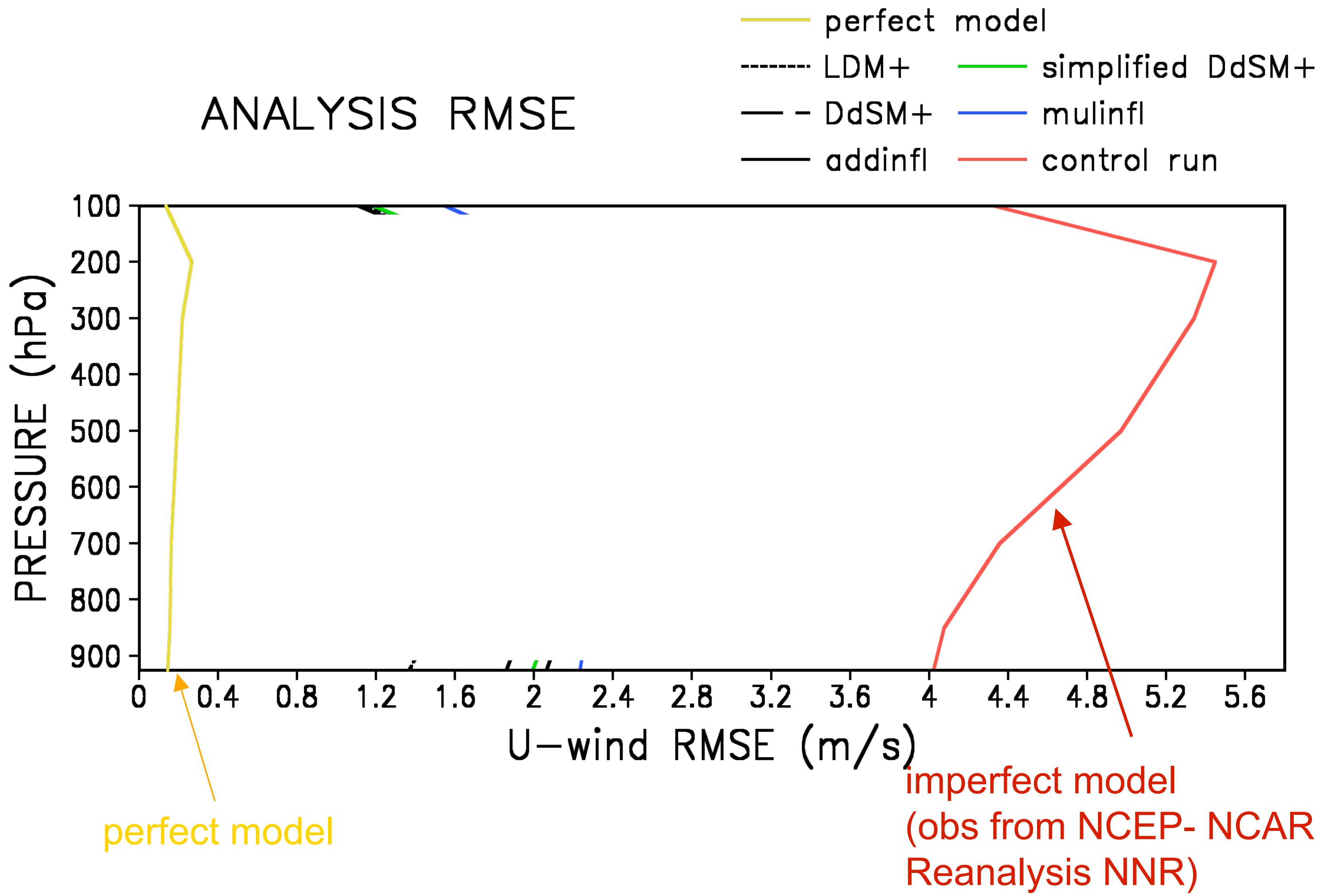
Motivation (2)

- However, most studies to date have tested EnKF systems under perfect model assumption with simulated observations. Only within the last few years have EnKF been tested in assimilating real observations.

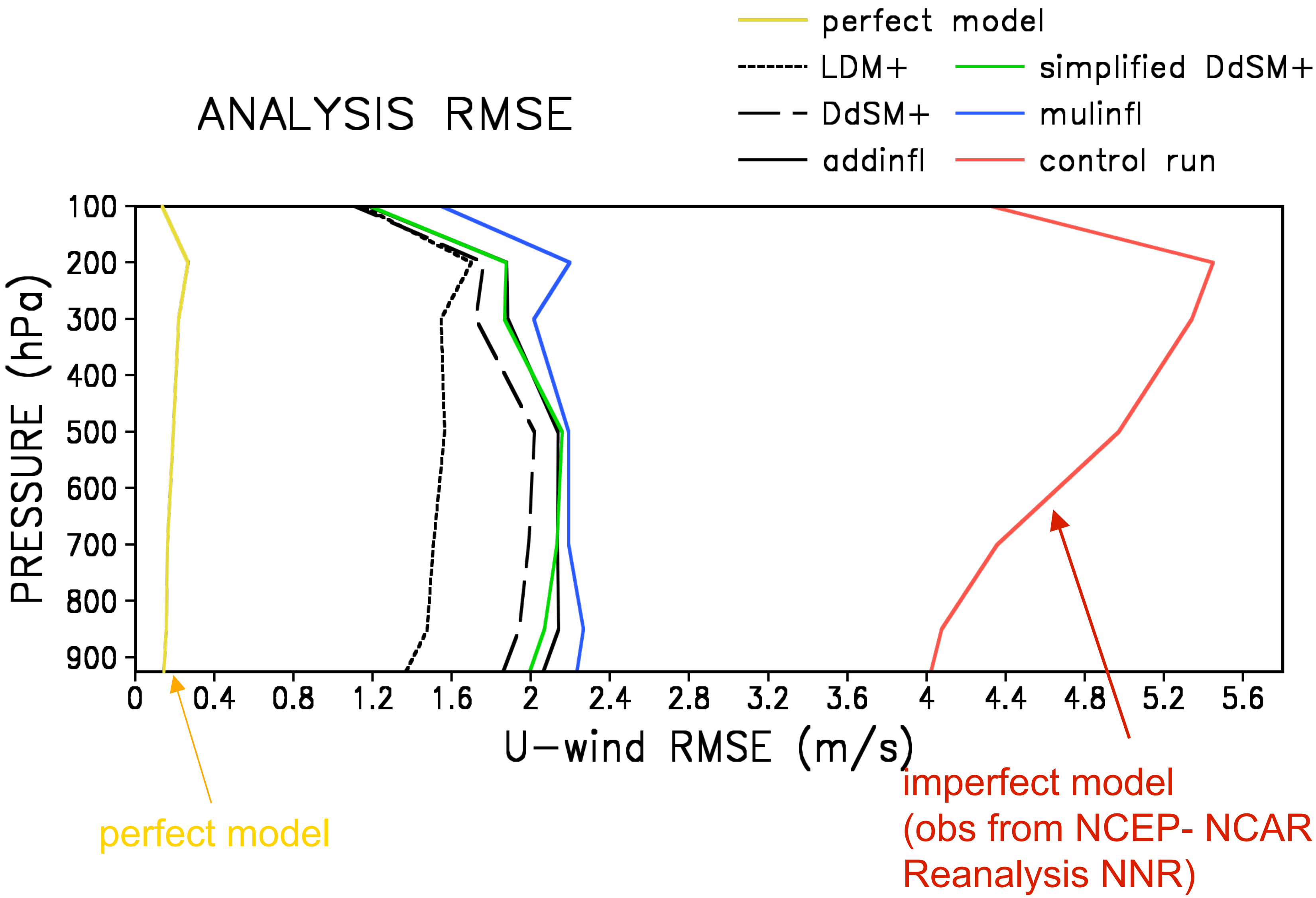
- In the **real-data** application, we have to deal with several issues, such as:
  1) model errors - THIS TALK
  2) inconvenience of manually tuning the inflation factor
  3) imperfect observation error statistics
  4) imperfect forward observation operator
  5) non-Gaussianity of forecast and observation errors, etc.

} ANOTHER TALK BY HONG LI (items 2–3)

If we assume a perfect model, we can grossly underestimate the errors



Summary slide: We compare several methods to handle model errors

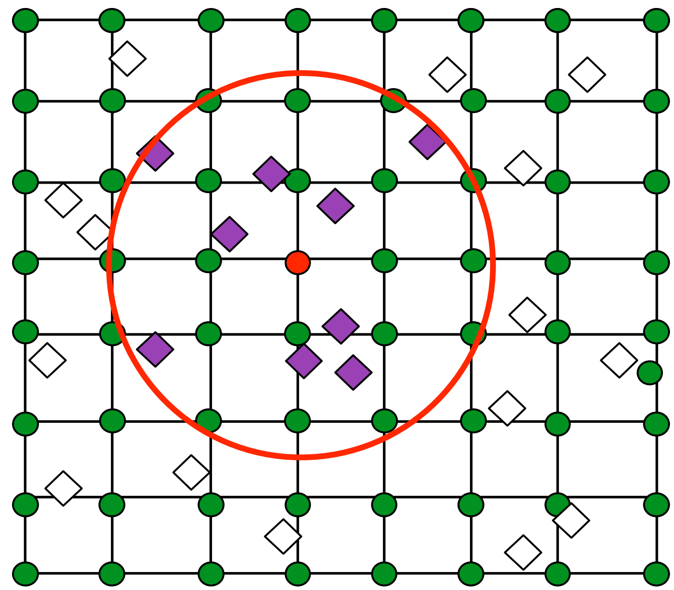


Local Ensemble Transform Kalman Filter (LETKF, Hunt et al. 2007)

- LETKF belongs to the family of EnKF, it is chosen as an representative of other EnKF systems.

- Performs Data Assimilation in local patch and matrix computations are done in ensemble space: both accurate and efficient, needs small ensemble.

- The analysis is computed independently at each grid point, could be highly parallel



Experimental Design

SPEEDY Model (Molteni 2003)

- Primitive equations, T30 L7 global spectral model

Data Assimilation: LETKF (30 ensemble members)

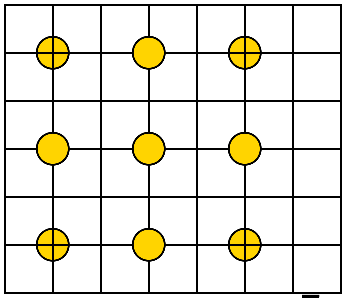
PERFECT MODEL:

Observations: from a SPEEDY "nature run" plus "random errors"

IMPERFECT MODEL:

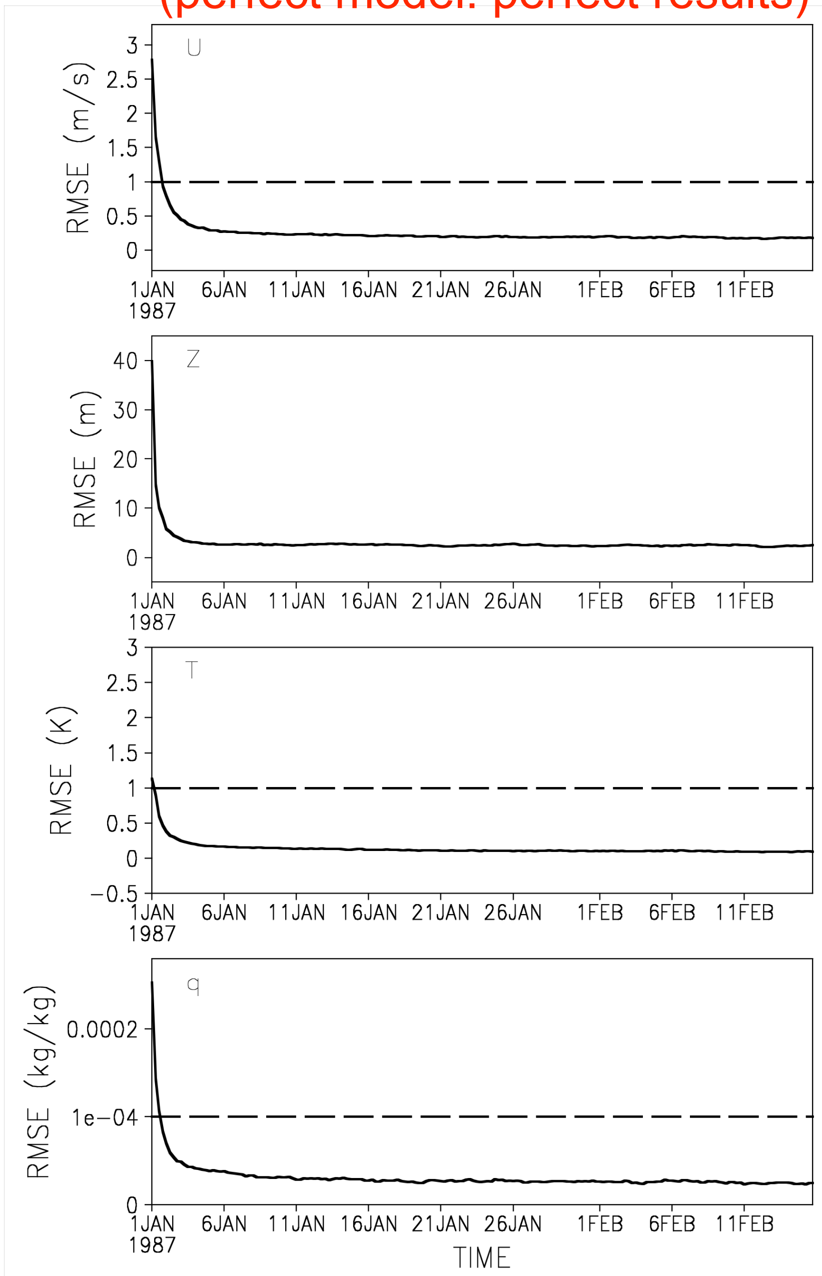
Observations: from the NCEP Reanalysis plus "random errors"

(assumes NNR approximates the real atmosphere, whereas SPEEDY has its own biased climatology.)



Analysis RMSE (500hPa)

(perfect model: perfect results)



30 ensemble members

Δ=0.05 multiplicative inflation

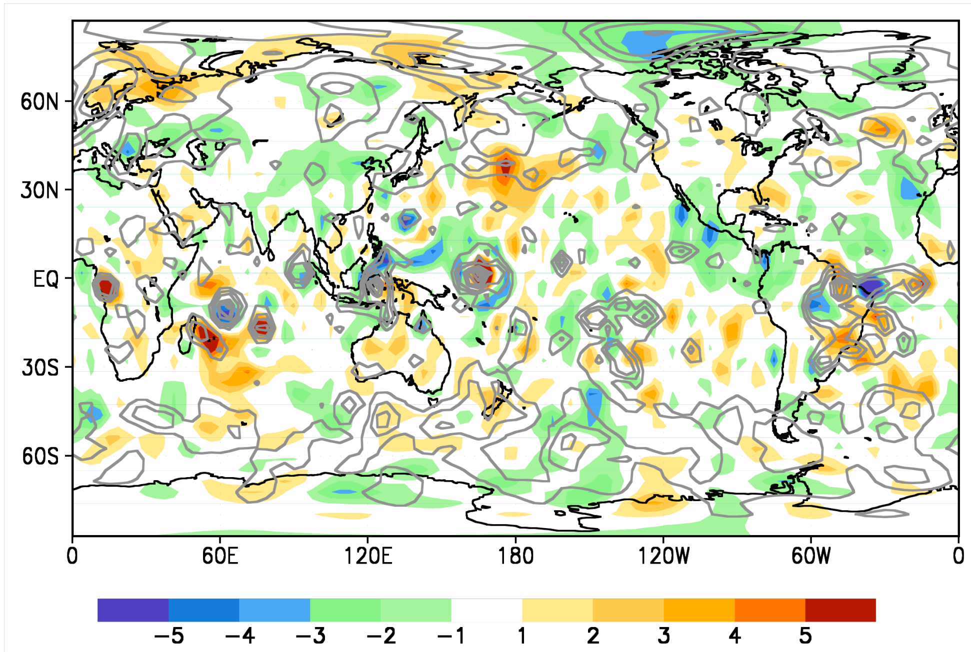
RMSE (solid line) are much smaller than the observational error standard deviations (dashed line)

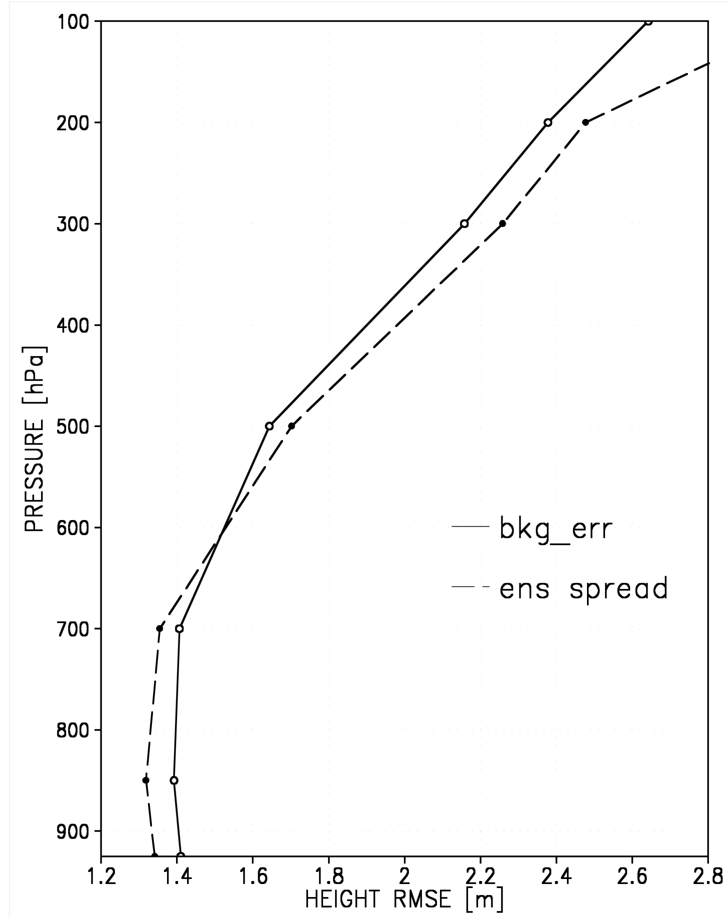
Perfect model

Background error (shades)

Ensemble spread (contour)

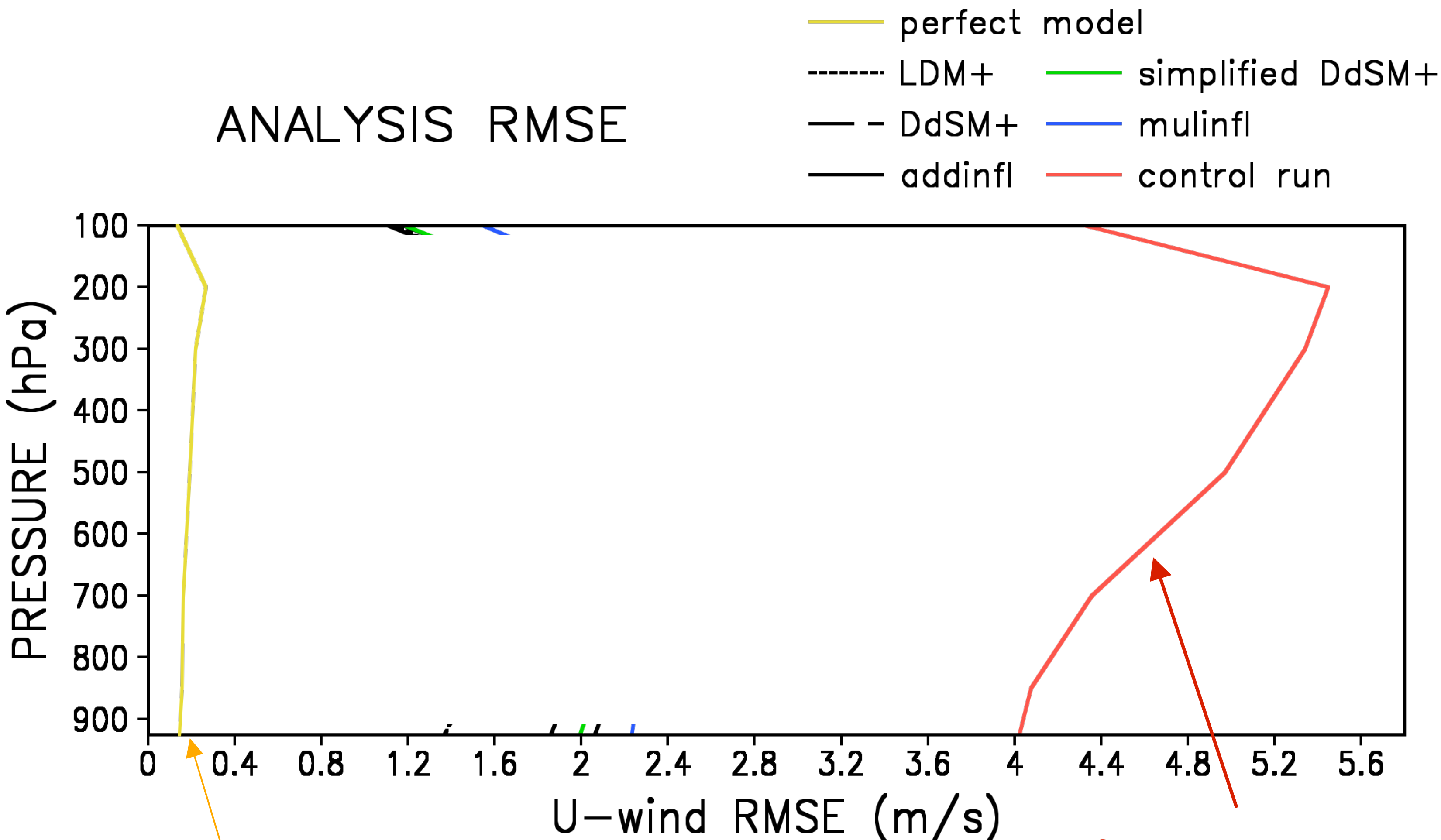
(500hPa heights)





LETKF ensemble spread captures the true background uncertainty in both structure and magnitude for perfect models, but…

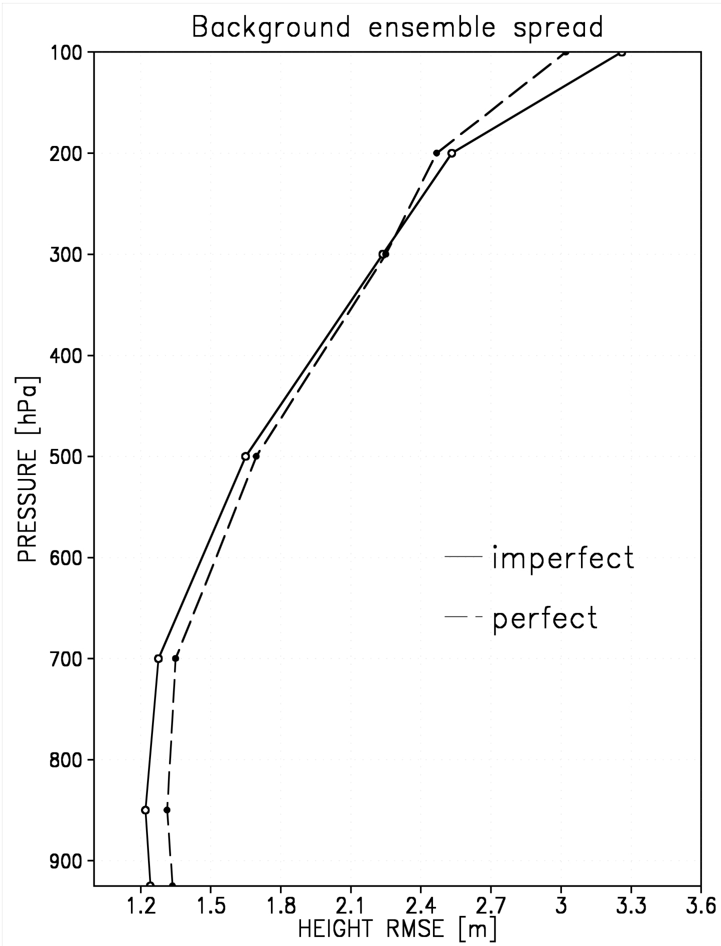
If we assume a perfect model, we can grossly underestimate the errors



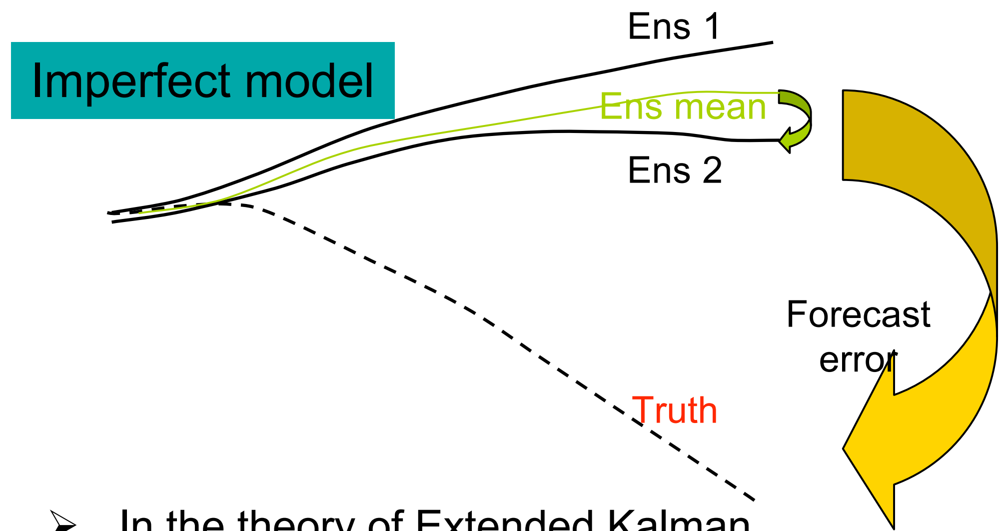
perfect model

imperfect model (obs from NCEP- NCAR Reanalysis NNR)

— Why is EnKF vulnerable to model errors ?



The ensemble spread is ‘blind’ to model errors



- In the theory of Extended Kalman filter, forecast error is represented by the growth of errors in IC and the model errors.

$$\mathbf{P}_i^f = \mathbf{M}_{\mathbf{x}_{i-1}^a} \mathbf{P}_{i-1}^a \mathbf{M}_{\mathbf{x}_{i-1}^a}^T + \mathbf{Q}$$

- However, in ensemble Kalman filter, error estimated by the ensemble spread can only represent the first type of errors.

$$\mathbf{P}_i^f \approx \frac{1}{k-1} \sum_{i=1}^{K} (x_i^f - \overline{x^f})(x_i^f - \overline{x^f})^T$$

How to deal with model errors in EnKF ?

Approaches:

1. Account for random model error in background error covariance (parameterize Q)

$$\mathbf{P}_i^f = \mathbf{M}_{\mathbf{x}_{i-1}^a} \mathbf{P}_{i-1}^a \mathbf{M}_{\mathbf{x}_{i-1}^a}{}^T + \mathbf{Q}$$

1.1 Multiplicative inflation
1.2 Additive inflation

2. Bias removal methods (estimate the model bias, then remove it)

$\widetilde{x}^f = x^f$ - estimated model bias

2.1 Dee and da Silva (1998) method (DdSM)

2.2 Baek et al. (2005) method

2.3 Low-dimensional method (LDM, Danforth et al. 2007)

Parameterized (random) model error covariance Q

1.1 Multiplicative inflation

$$\mathbf{P}_i^f = \frac{1}{k-1}\sum_{i=1}^{K}(x_i^f - \overline{x^f})(x_i^f - \overline{x^f})^T$$

Q_{multi} is in the same subspace as P^f

$$\widetilde{\mathbf{P}}_i^f = (1+\Delta) * \mathbf{P}_i^f = \mathbf{P}_i^f + \Delta \mathbf{P}_i^f$$

$\Delta \mathbf{P}_i^f \rightarrow \mathbf{Q}$

1.2 Additive inflation

$\widetilde{x}_i^f = x_i^f + r$ *(randomly selected NNR 6hour tendency)

$$\widetilde{\mathbf{P}}_i^f = \frac{1}{k-1}\sum_{i=1}^{K}(\widetilde{x}_i^f - \overline{\widetilde{x}^f})(\widetilde{x}_i^f - \overline{\widetilde{x}^f})^T$$

$$\widetilde{\mathbf{P}}_i^f = \mathbf{P}_i^f + \mathbf{Q}$$

Q_{addit} explores unstable directions outside P^f

Bias removal schemes (1)

b=bias

2.1 DdSM (Dee and da Silva,1998)

$$b_t^f = \mu b_{t-1}^a$$

$$b^a = b^f - K_b[y^o - (Hx^f) - Hb^f)]$$

$$K_b = P^b H^T (HP^b H^T + HP^f H^T + R)^{-1}$$

$$\tilde{x}^f = x^f - b^a$$

$$x^a = \tilde{x}^f + K_x[y^o - H\tilde{x}^f]$$

$$K_x = P^f H^T (HP^f H^T + R)^{-1}$$

Do data assimilation twice:

first for model bias

$$P^b = \alpha * P^f \qquad 0 < \mu, \alpha \le 1$$

need to be tuned

then for model state (expensive)

DdSM simplified version: (Radakovich et al 2001)

$$K_b = P^b H^T (HP^b H^T + HP^f H^T + R)^{-1}$$

$$= \alpha * P^f H^T ((1+\alpha) HP^f H^T + R)^{-1}$$

$$\approx \alpha * P^f H^T (HP^f H^T + R)^{-1}$$ (Assuming α is very small)

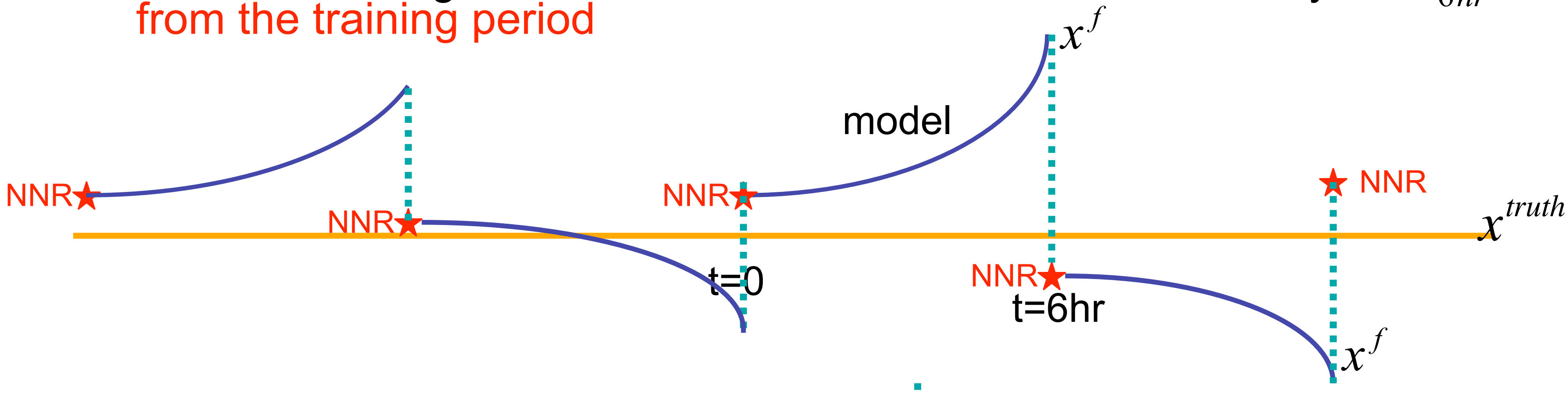
$$\Rightarrow K_b = \alpha * K_x \Rightarrow \delta b^a = \alpha * \delta x^a$$

$$b^a = \mu b_{t-1}^a - \alpha * \delta x^a$$

Bias removal schemes (Low Dimensional Method)

2.3 Low-dim method (Danforth et al, 2007: Estimating and correcting global weather model error. *Mon. Wea. Rev, J. Atmos. Sci., 2007*)

- Generate a long time series of model forecast minus reanalysis x^e_{6hr} from the training period



We collect a large number of estimated errors and estimate from them bias, etc.

$$\varepsilon^f_{n+1} = \mathbf{x}^f_{n+1} - \mathbf{x}^t_{n+1} = M(\mathbf{x}^a_n) - M(\mathbf{x}^t_n) + \mathbf{b} + \sum_{l=1}^{L}\beta_{n,l}\mathbf{e}_l + \sum_{m=1}^{M}\gamma_{n,m}\mathbf{f}_m$$

Forecast error due to error in IC

Time-mean model bias

Diurnal model error

State dependent model error

Low-dimensional method

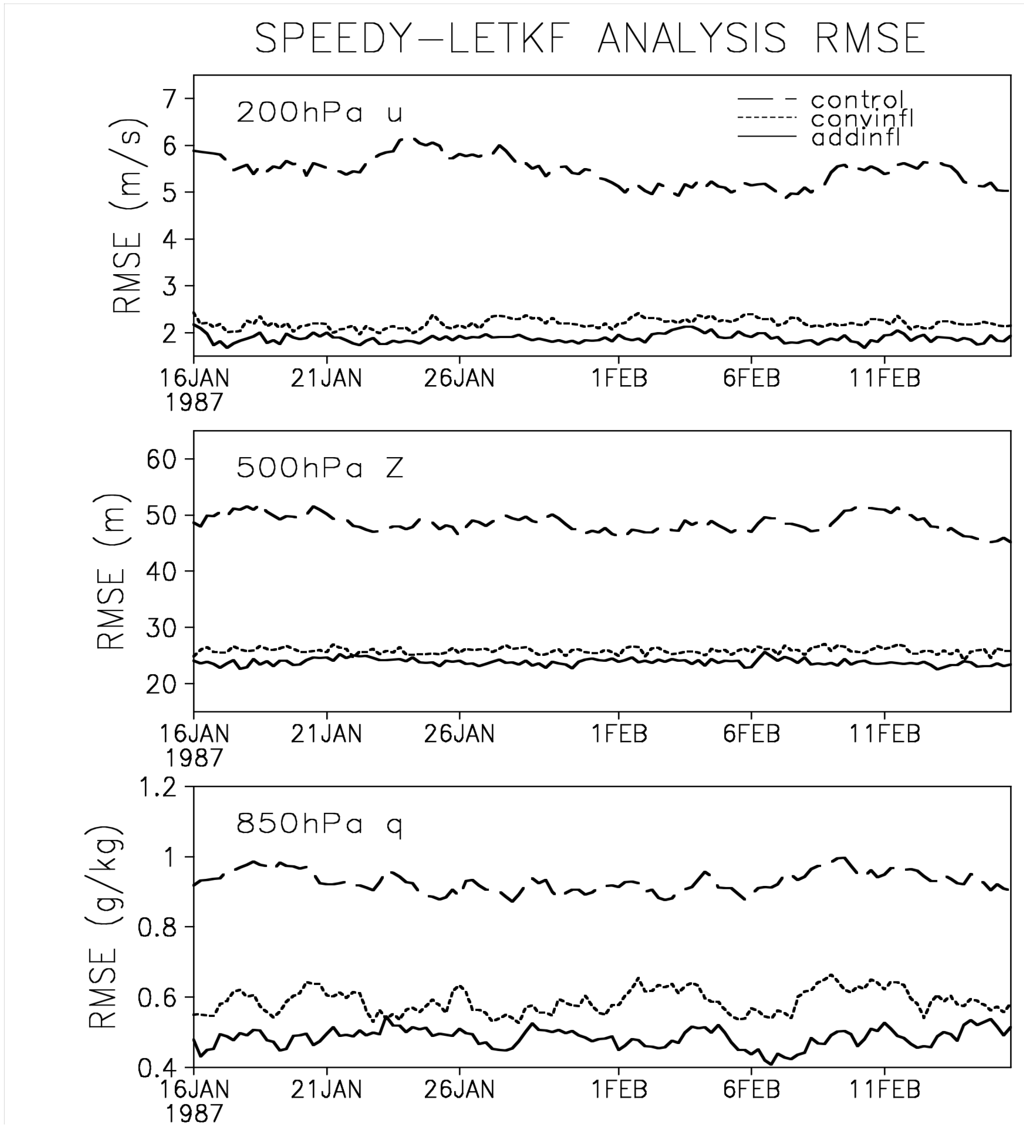
Include Bias, Diurnal and State-Dependent model errors:

$$\text{model error} = \mathbf{b} + \sum_{l=1}^{2} \beta_{n,l}\mathbf{e}_l + \sum_{m=1}^{10} \gamma_{n,m}\mathbf{f}_m$$

Having a large number of estimated errors allows to estimate the global model error beyond the bias

Results applying different methods to deal with model errors

multiplicative .vs. additive inflation



- . . Control run (Δ =0.05)

...... Multi inflation (Δ =1.5)

— Additive inflation (r =1.5)

Additive inflation is better than multiplicative inflation (it explores a larger subspace of errors)

DdSM .vs. additive inflation

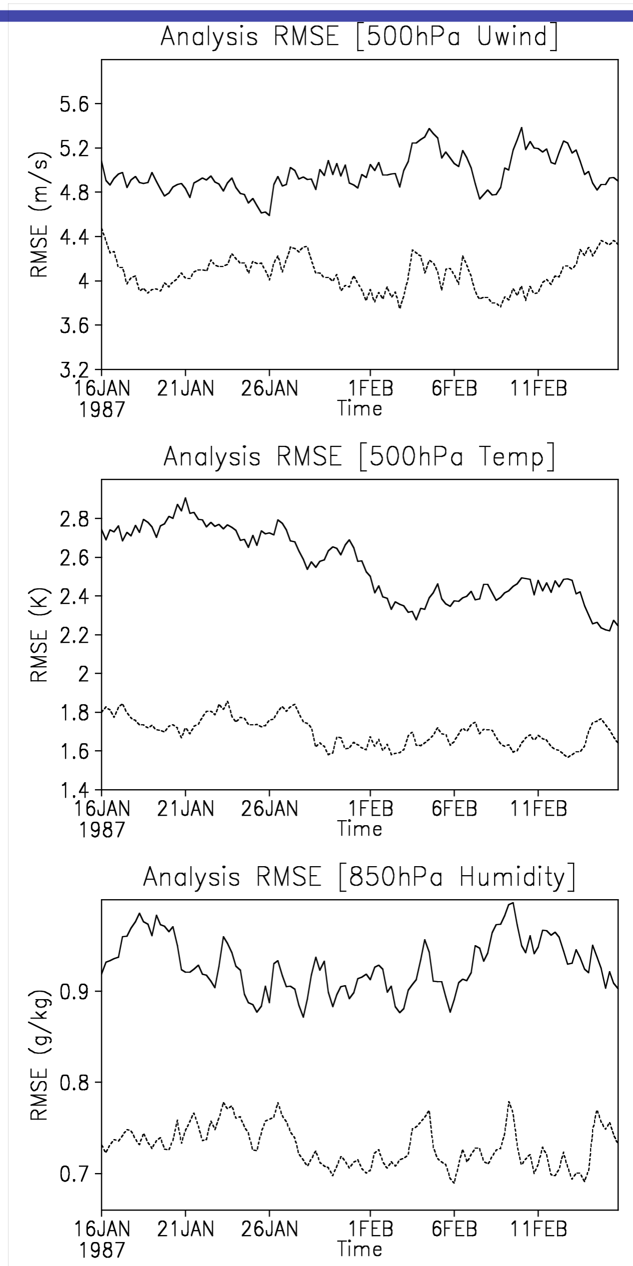
$$P^b = \alpha * P^f$$

Analysis RMSE (500 hPa Height)

	α	0.0	0.25	0.50	0.75	1.00
Pure DdSM	$r=0$			37.1	35.0	33.9
DdSM + additive inflation r	$r=0.25$		22.5	22.0	18.9	19.2
	$r=0.5$		19.8	17.6	20.4	20.1
	$r=0.6$			17.3		
Pure Additive inflation	$r=1.5$	23.8				

- Model error = model bias + random model error
- DdSM is designed to correct model bias, it can not correct random model error
- After accounting for random model error, DdSM+ outperforms additive inflation

LDM performance (1) – time-independent bias correction



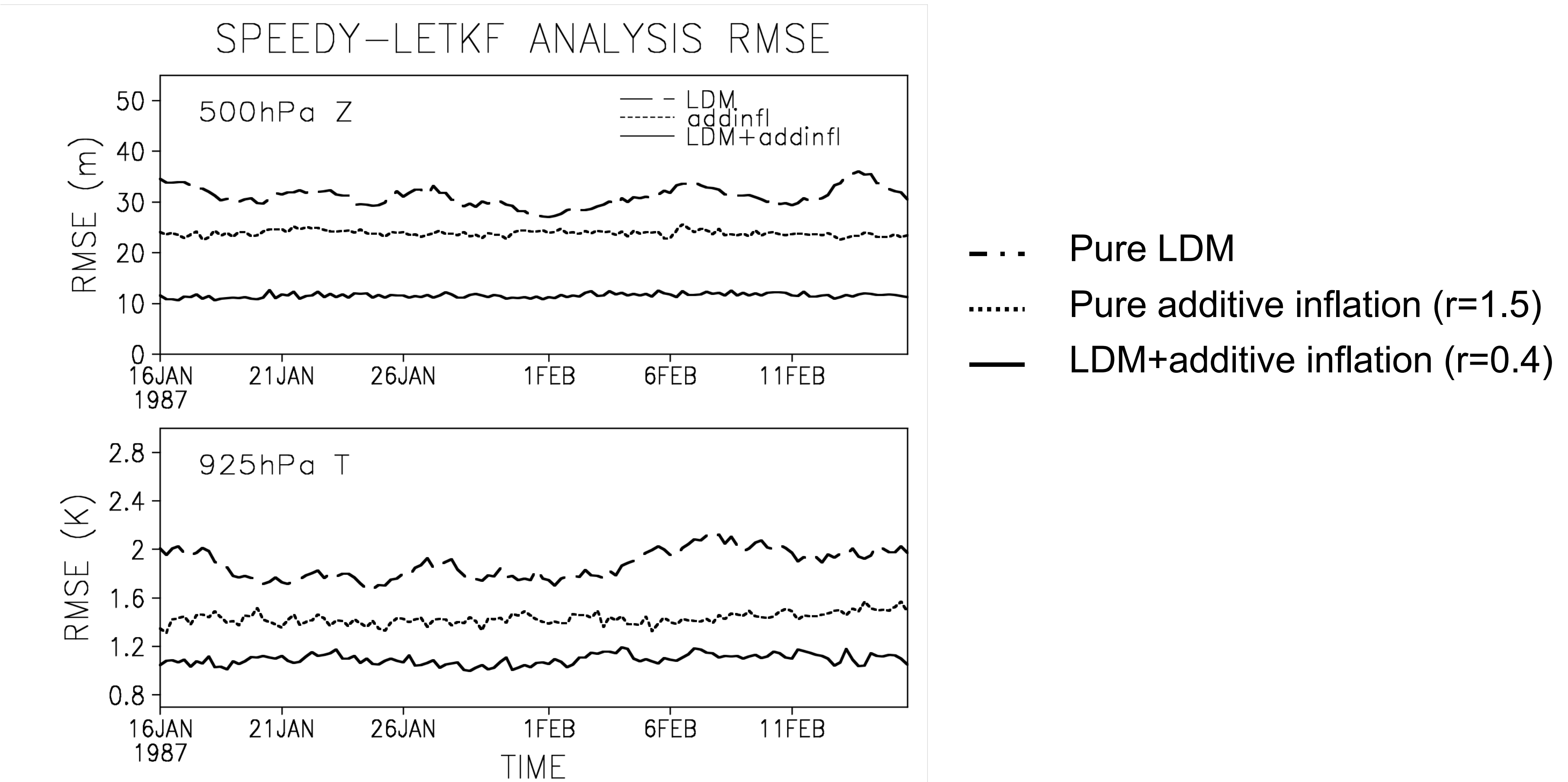
$$\text{model error} = \mathbf{b} + \cancel{\sum_{l=1}^{L} \beta_{n,l} \mathbf{e}_l} + \cancel{\sum_{m=1}^{M} \gamma_{n,m} \mathbf{f}_m}$$

$$\tilde{x}^f = x^f - \mathbf{b}$$

Control run

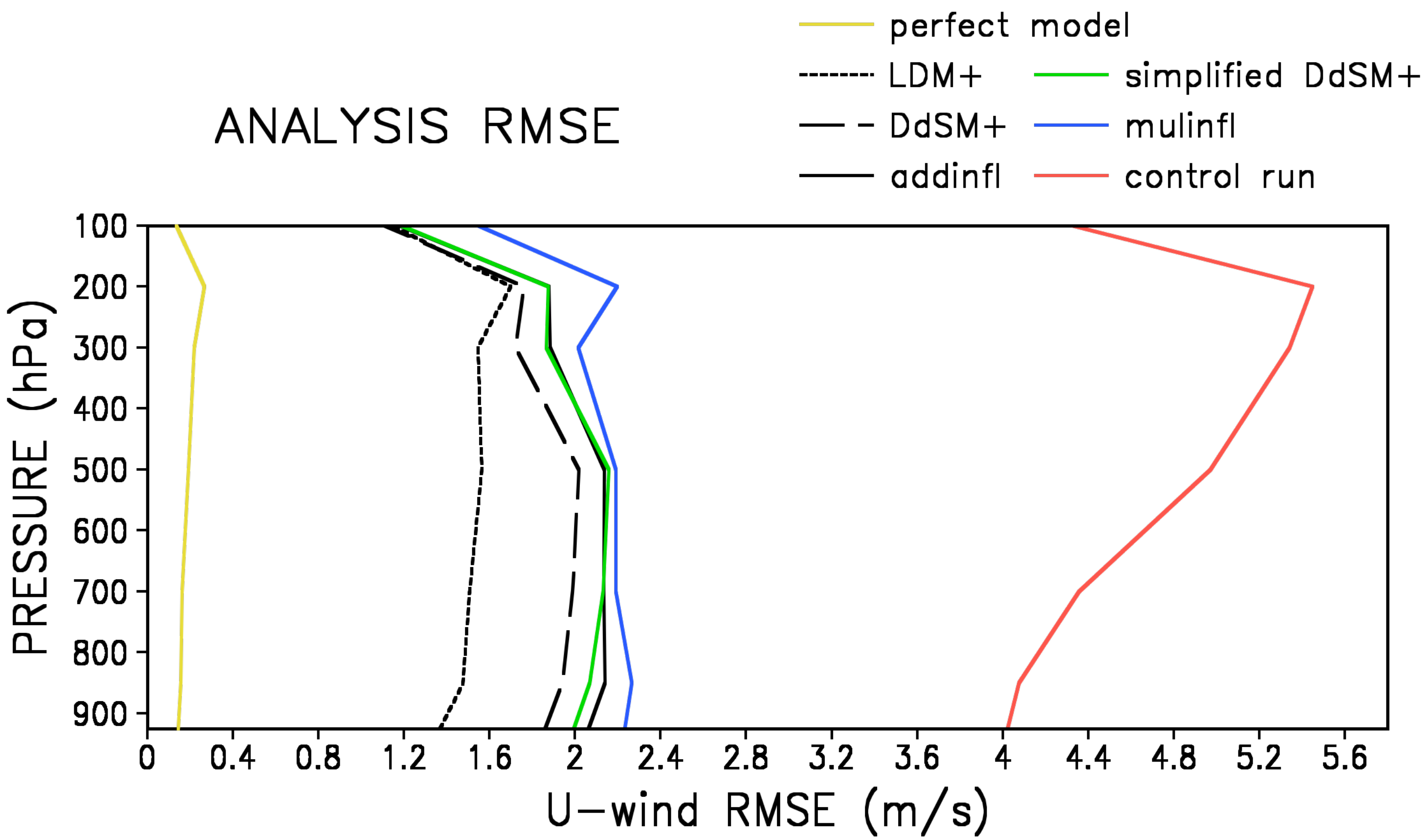
LDM

Low-Dim Method .vs. additive inflation



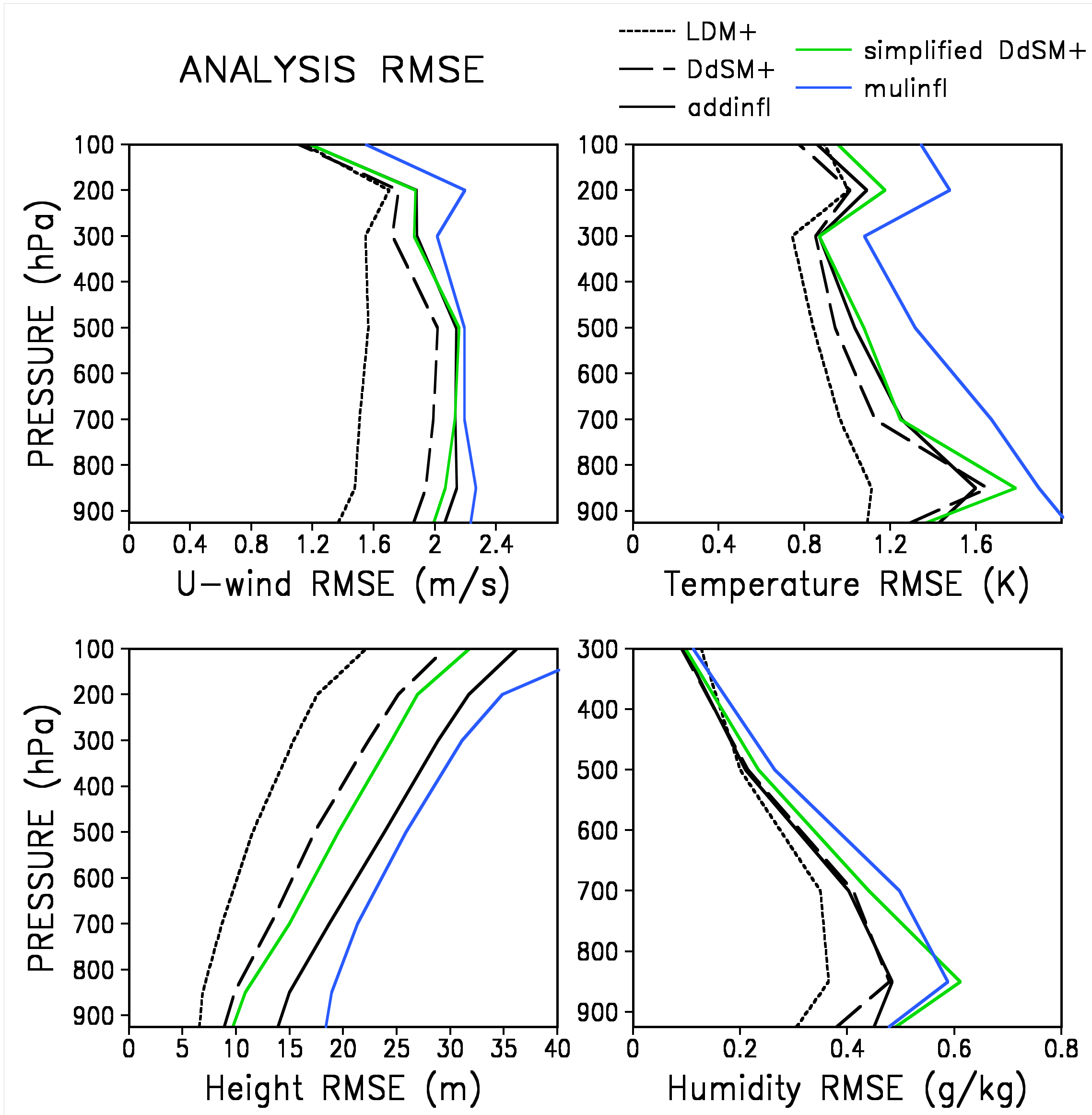
- Pure LDM cannot beat additive inflation since it can not correct random model error
- LDM+, the combination of LDM (systematic errors) and additive inflation (random errors) outperforms additive inflation scheme alone

Comparison (Analysis RMSE)

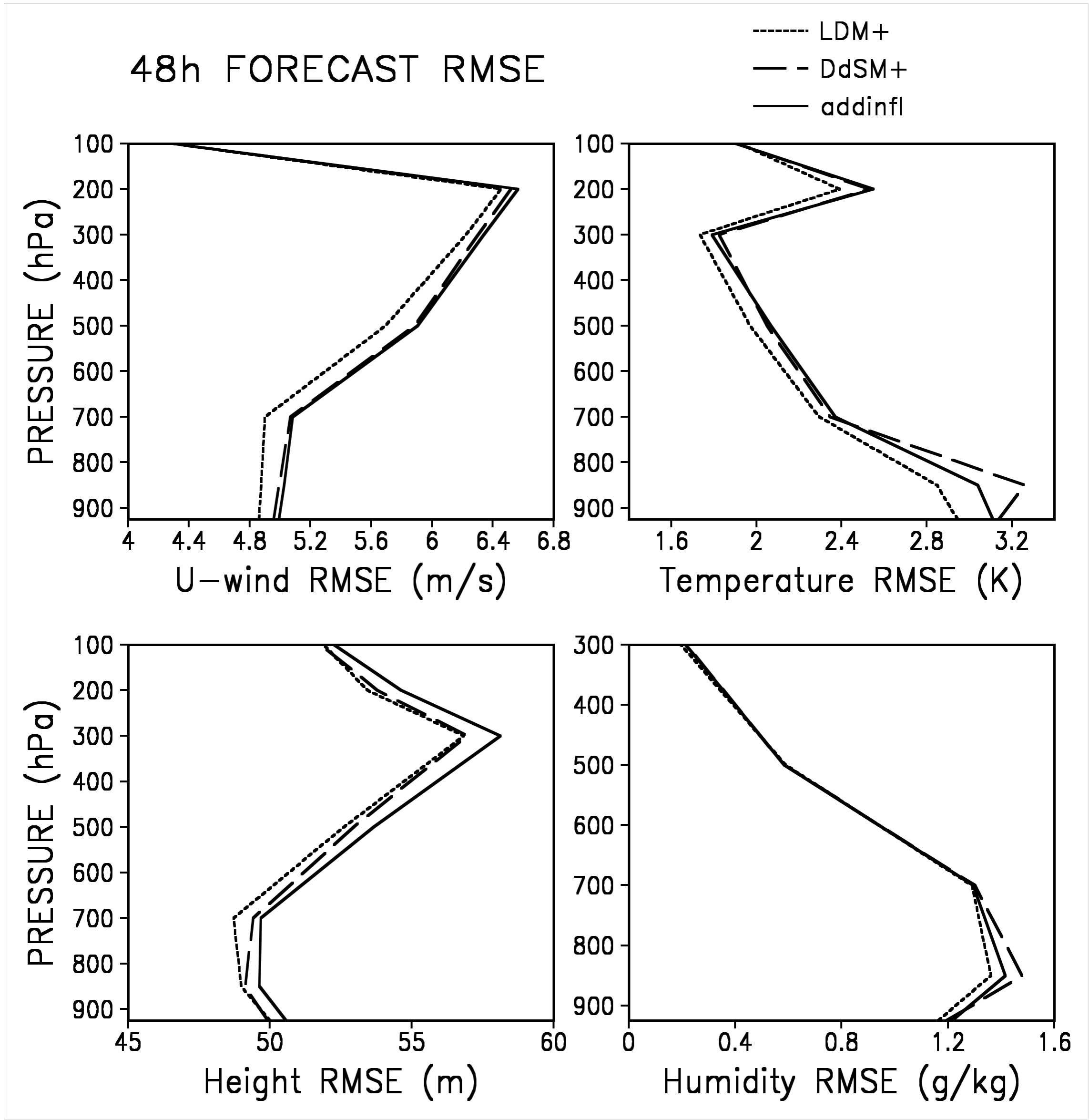


- All methods have made huge improvements compared to the 'control run'
- LDM+, DdSM+ outperform two inflation schemes
- Of all methods, the LDM+ provides the best analysis

Comparison (Analysis RMSE)



Comparison (48-hour Forecast RMSE)



The advantage of the LDM+ over the other two methods decreases due to the growth of unstable errors. However, it is still significantly better.

Summary

1) The LETKF works very well in perfect model scenario.
2) Without correcting model errors or at least accounting for their effects, the performance of the LETKF in the presence of model errors is rather poor due to the fact that the ensemble spread is ‘blind’ to model errors.
3) Model errors include model biases and random noise. The pure bias correction schemes (Dee and daSilva and Low-Dim Method) are slightly worse than the inflation schemes (random errors).
4) After accounting for random noise, the bias correction methods (DdSM+ and LDM+) are generally superior to any of the inflation methods.
5) Of these methods, LDM+ gives the best results. The analyses are more accurate and less biased.

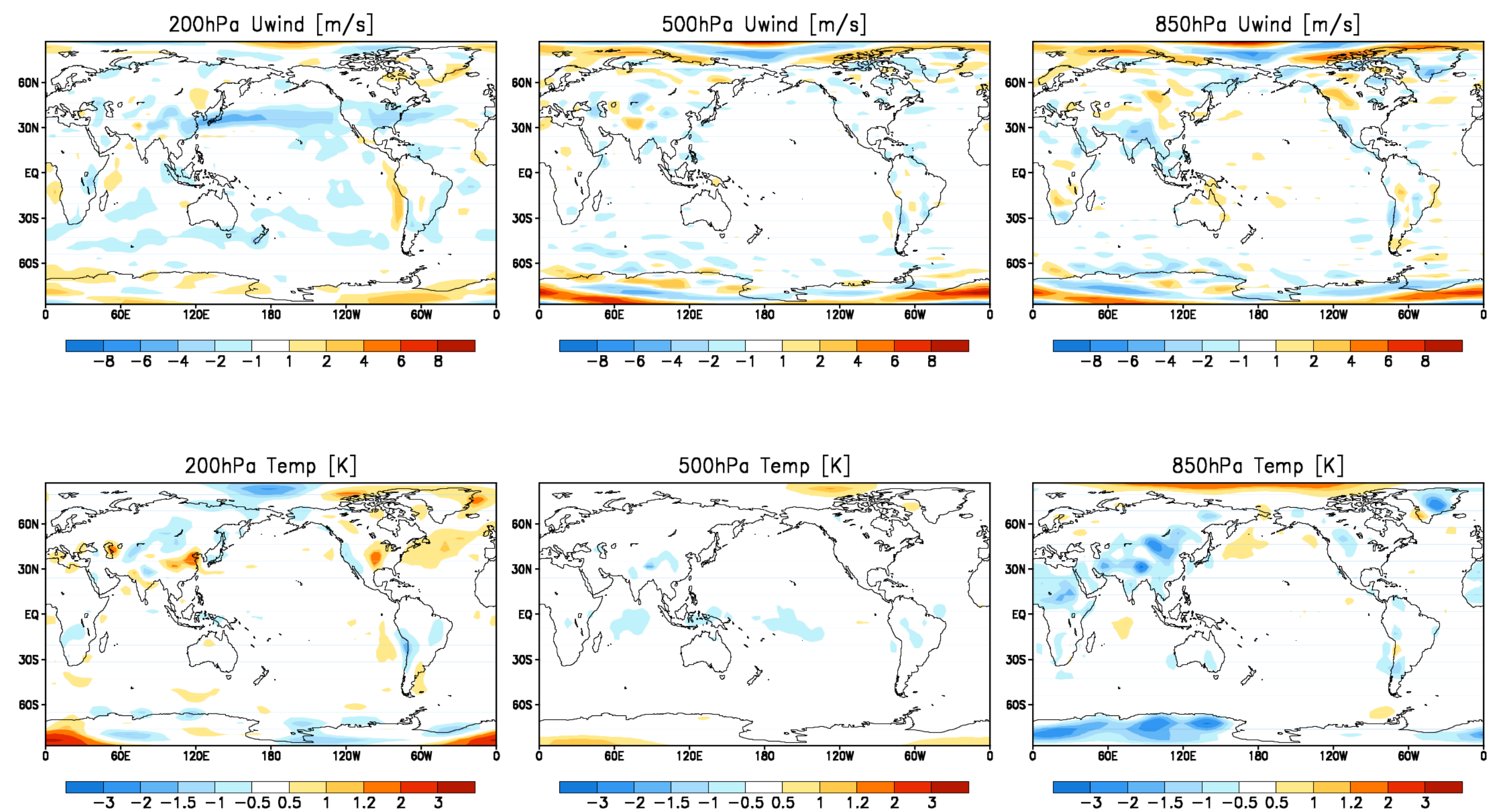
SPEEDY 6-hour model errors against NNR (bias)

1987 Jan 1~ Feb 15

Mean error (bias)

U-WIND

TEMP



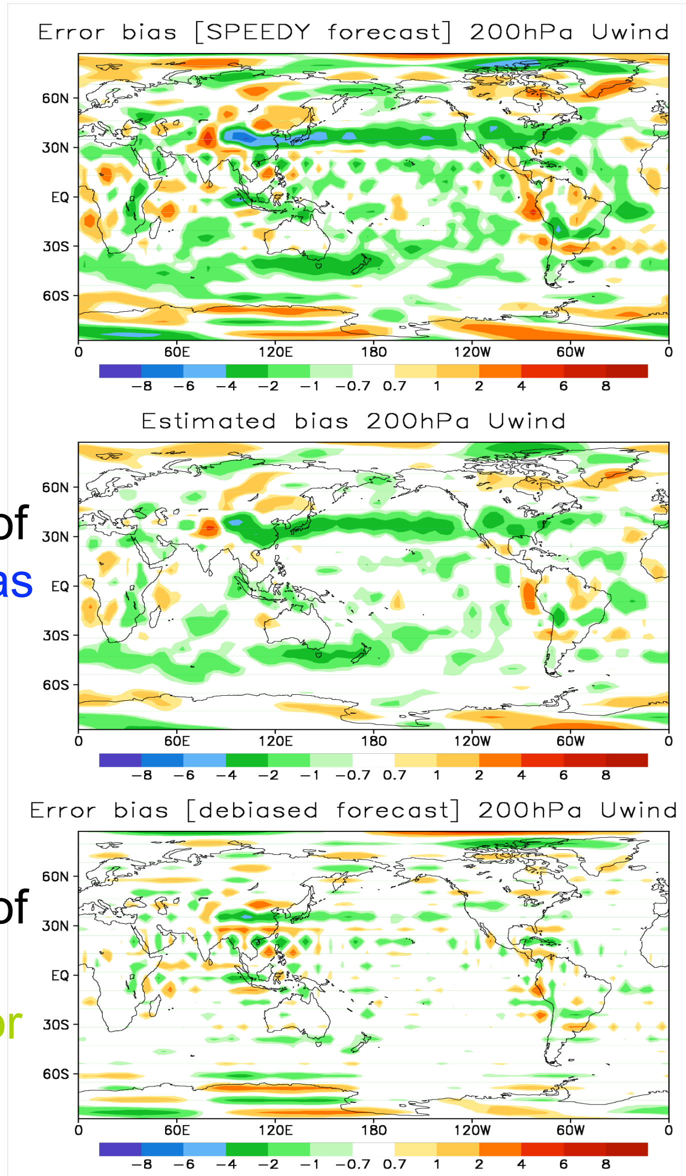
DdSM+ performance

DdSM +additive inflation (r=0.6)

Time-mean of biased forecast

Time-mean of estimated bias

Time-mean of 'debiased' forecast error



- In general, the estimated bias field captures the structure of the forecast bias well
- After subtracting the estimated bias from the original biased forecast, the 'debiased' forecast exhibits significantly less bias.
- But in-between observations there is a leftover bias in the Dee and daSilva scheme plus additive inflation.

SPEEDY 6 hr model errors against NNR (diurnal cycle)

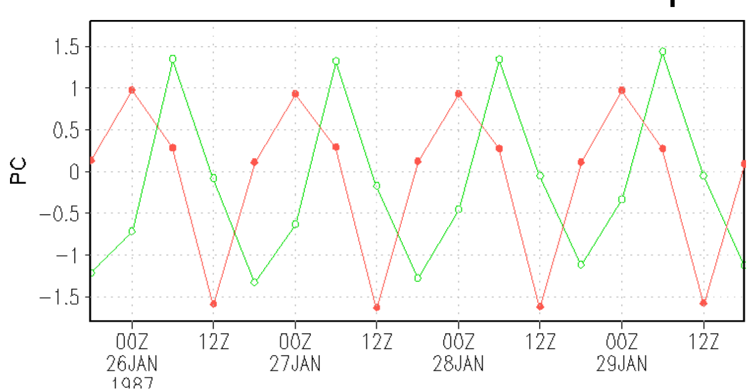
1987 Jan 1~ Feb 15

Error anomalies

$$x^{e}_{6hr(i)}{}' = x^{e}_{6hr} - \overline{x^{e}_{6hr}}$$

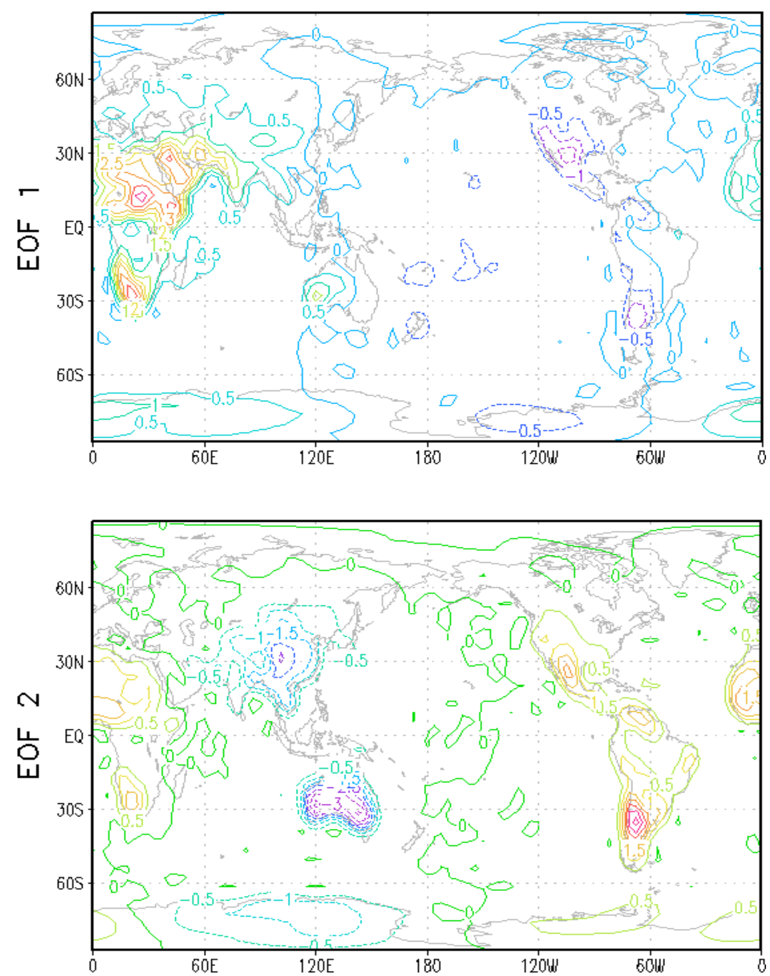
pc1

pc2

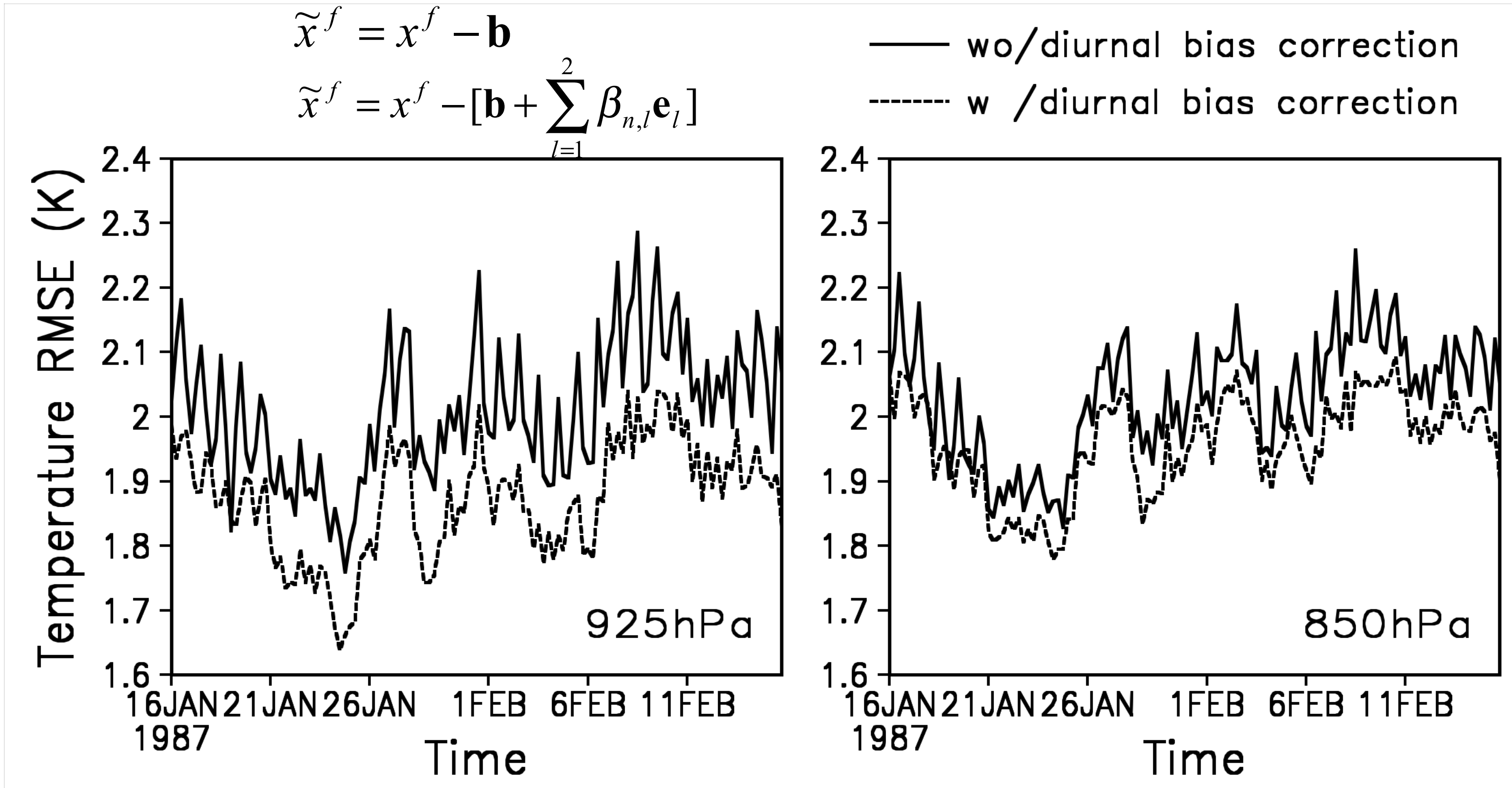


- For temperature at lower-levels, in addition to the time-independent bias, SPEEDY has diurnal cycle errors because it lacks diurnal radiation forcing

Leading EOFs for 925 mb TEMP



LDM performance (2) – diurnal bias correction

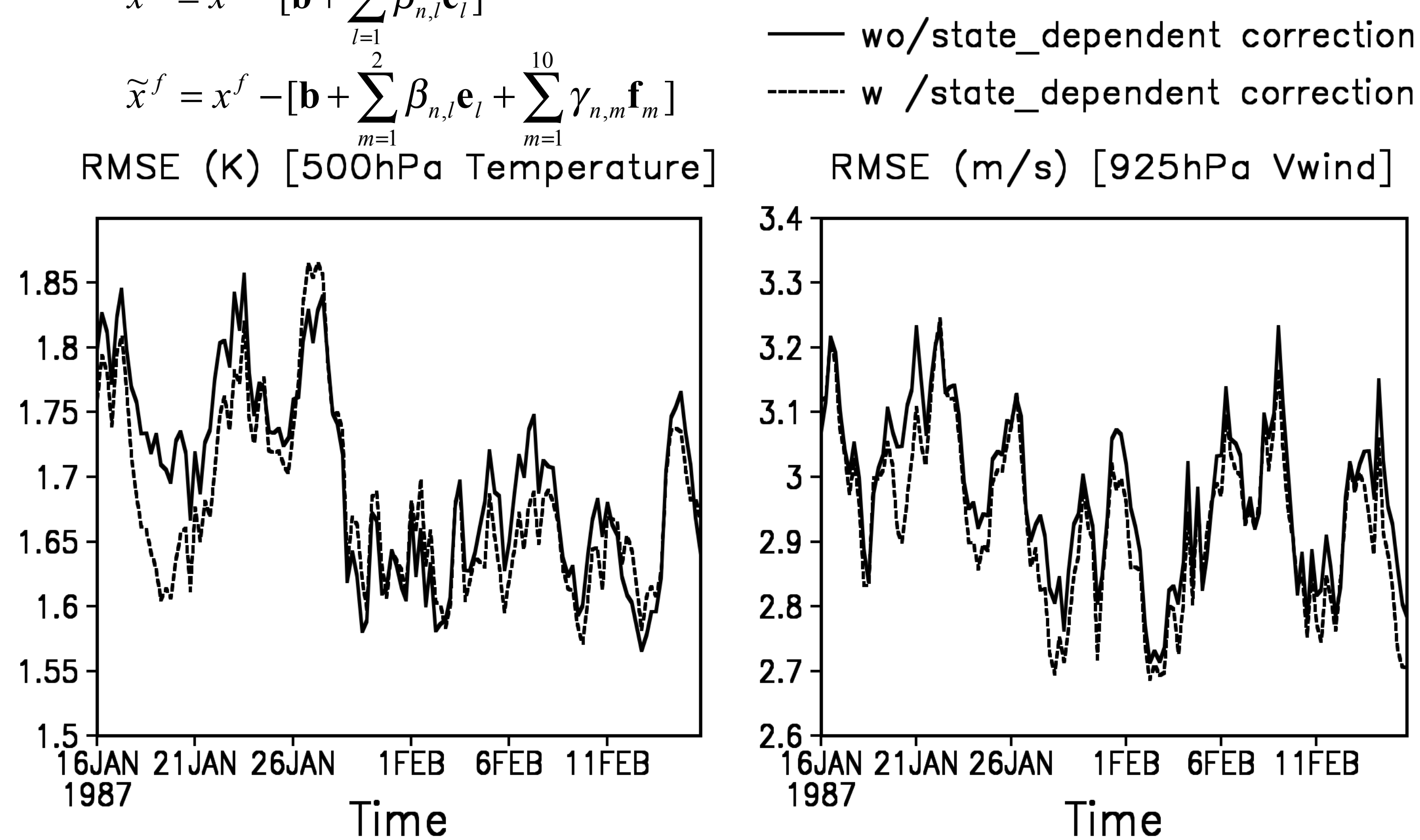


➢ Without diurnal bias correction, the analysis RMSE is higher and has strong diurnal variability.

LDM performance (3) – state-dependent error correction

$$\tilde{x}^f = x^f - [\mathbf{b} + \sum_{l=1}^{2} \beta_{n,l}\mathbf{e}_l]$$

$$\tilde{x}^f = x^f - [\mathbf{b} + \sum_{m=1}^{2} \beta_{n,l}\mathbf{e}_l + \sum_{m=1}^{10} \gamma_{n,m}\mathbf{f}_m]$$



➢ The impact of the state-dependent error correction on the global-averaged analysis RMSE is generally small but overall positive.